Math 104B, Winter 2019

Number Theory II, HW 9

Due Thursday March 14th by 10AM in Shubham Sinha's box.

From Niven, Zuckerman, Montgomery (5th Ed.):

- Problems (Section 7.7, page 351):

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- Problems (Section 7.8, page 356):

  2, 5, 7, 8, 9, 11

Problem A. Suppose $\alpha \in \mathbb{R}$ is given by a purely periodic continued fraction

$$\alpha = \langle \overline{a_0, a_1, \dots, a_N} \rangle.$$

Let $\beta = \langle \overline{a_N, a_{N-1}, \dots, a_0} \rangle$ be its reflection (where the partial quotients appear in the opposite order).

(a) Verify that α and $-\frac{1}{\beta}$ satisfy the **same** quadratic equation. (`Hint`: Here exercise 4 on page 333 in section 7.3 is helpful, cf. HW8.)

(b) Using that both α and β are necessarily reduced, rule out that $-\frac{1}{\beta} = \alpha$ and conclude that $-\frac{1}{\beta} = \bar{\alpha}$ (=the conjugate of α in the pertaining quadratic field).

(c) Let $D > 0$ be a non-square integer with continued fraction

$$\sqrt{D} = \langle b_0, \overline{b_1, \dots, b_M, 2b_0} \rangle.$$

By applying (b) to $\alpha = b_0 + \sqrt{D}$ deduce that $\sqrt{D}$ has a **palindromic** continued fraction – meaning that

$$b_1 = b_M, \quad b_2 = b_{M-1}, \quad b_3 = b_{M-2}, \quad \dots.$$

Problem B. Thank you for a great quarter! Please do us a big favor and fill out your CAPE teaching evaluations (due Monday March 18th at 8AM).