Math 109, Spring 2018

Mathematical Reasoning , HW 4

Due May 1st by 10AM in Roman Kitsela's box

From Hammack's `Book of Proof`:

- Exercises 2.4 (–): *2, 4*
- Exercises 2.5 (–): *2, 4, 6*
- Exercises 2.6 (A): *4, 8*
- Exercises 2.6 (B): *12, 14*
- Exercises 2.7 (–): *2, 4, 6, 8, 10*

Problem I. Let $(x_n)_{n\in\mathbb{N}}$ be a sequence $x_1, x_2, x_3, \ldots$ of real numbers. The sequence is said to be *Cauchy* if the following condition holds.

$$\forall \epsilon > 0 \ \ \exists N \in \mathbb{N} \ \ \forall m, n \geq N : \ \ |x_m - x_n| < \epsilon.$$

(a) Write out this definition in plain English.

(b) Using quantifiers state what it means for $(x_n)_{n\in\mathbb{N}}$ to not be Cauchy.

(c) Show that the following two statements are true:

(1) $(x_n)_{n\in\mathbb{N}}$ is convergent $\Longrightarrow (x_n)_{n\in\mathbb{N}}$ is Cauchy

(2) $(x_n)_{n\in\mathbb{N}}$ is Cauchy $\Longrightarrow (x_n)_{n\in\mathbb{N}}$ is bounded.

(d) Give an example of a bounded sequence which is not Cauchy.

(e) Can you think of a Cauchy sequence which is not convergent?