Math 109, Spring 2018

Mathematical Reasoning , HW 5

Due May 8th by 10AM in Roman Kitsela's box

From Hammack's `Book of Proof`:

- Exercises 2.9 (–): *4*
- Exercises 2.10 (–): *4*
- Exercises 3.2 (–): *4, 6, 8*
- Exercises 3.3 (–): *2, 4, 10, 14*
- Exercises 3.4 (–): *2, 4, 12, 14*

Problem I. Consider an $m \times n$ grid in the plane. How many shortest paths are there from the lower left corner to the upper right corner? (Here *paths* move along the grid lines. This is known as Manhattan geometry.)

Problem II. Recall that the *well-ordering principle* for $\mathbb{N}$ says that

(WOP) Every non-empty subset of $\mathbb{N}$ has a smallest element.

(a) Write out with symbolic logic ($\in$, $\wedge$, $\forall$ etc.) what it means for an element $s \in A$ to be a smallest element of a subset $A \subseteq \mathbb{N}$.

(b) Prove that this smallest element s is necessarily uniquely determined (for an arbitrary but fixed non-empty subset $A \subseteq \mathbb{N}$).

(c) Using only the (WOP) show that the following statement is true[1]:

Every non-empty bounded subset of $\mathbb{N}$ has a largest element.

`Hint`: Apply the well-ordering principle to the set of bounds b for $X \subseteq \mathbb{N}$.

[1]A subset $X \subseteq \mathbb{N}$ is *bounded* if there is a $b \in \mathbb{N}$ such that $x \leq b$ holds for all $x \in X$.