MATH 109, SPRING 2018

MATHEMATICAL REASONING , HW 7

Due May 22nd by 10AM in Roman Kitsela's box

From Hammack's `Book of Proof`:

- Exercises Sect. 11.4 (p. 194): *2, 4, 6, 8*
- Exercises Ch. 10 (p. 169): *2, 4, 6, 8, 10, 12, 16, 20, 24*

Problem I.

(a) Describe the partition of $\mathbb{Z}$ resulting from the relation $\equiv$ (mod 2).

(b) Consider the partition of $\mathbb{Z}$ below:

$$\mathbb{Z} = \{\cdots -6, -3, 0, 3, 6 \cdots\} \cup \{\cdots -5, -2, 1, 4, 7 \cdots\} \cup \{\cdots -4, -1, 2, 5, 8 \cdots\}.$$

Which familiar relation on $\mathbb{Z}$ does it arise from?

(c) Which of the three subsets in (b) contains the number 2018?

Problem II.

(a) Find all positive integers $x < 100$ such that $x \equiv 2018 \pmod{13}$.

(b) Find all positive integers $x < 100$ such that $9x \equiv 1 \pmod{13}$.

(c) Does the residue class $[9]$ have an inverse in $\mathbb{Z}_{13}$? If so exhibit the inverse residue class as $[x]$ for some non-negative integer $x < 13$.

(d) Does the residue class $[9]$ have an inverse in $\mathbb{Z}_{18}$? If so exhibit the inverse residue class as $[x]$ for some non-negative integer $x < 18$.