Mathematical Reasoning , HW 8

Due May 29th by 10AM in Roman Kitsela's box

From Hammack's `Book of Proof`:

- Exercises <u>Ch. 10</u> (p. 169): *26, 30, 34*

- Exercises <u>Sect. 11.0</u> (p. 178): *2, 4, 10, 14*

- Exercises <u>Sect. 11.1</u> (p. 182): *4, 10, 14*

- Exercises <u>Sect. 11.2</u> (p. 187): *2, 6*

- Exercises <u>Sect. 11.3</u> (p. 190): *2, 4*

Problem I. Define a relation $\sim$ on $\mathbb{R}$ by declaring that $(\forall a, b \in \mathbb{R})$

$$a \sim b \iff a - b \in \mathbb{Z}.$$

(a) Check that $\sim$ is an equivalence relation on $\mathbb{R}$.

(b) Give a geometric description of the equivalence class $[a]$ containing $a \in \mathbb{R}$.

(c) For any given $a \in \mathbb{R}$ prove that there is a unique $r \in \mathbb{R}$ for which

$$a \sim r \ \wedge \ 0 \leq r < 1.$$