Math 109, Winter 2020

Mathematical Reasoning, HW 0

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From the book by Chartrand, Polimeni, and Zhang:

- Exercises (Section 1.1, pages 17–18):

  1.4, 1.6

Problem A. Read Chapter 0 from the book on Communicating Mathematics.

Problem B. Try some of the *odd*-numbered exercises in sections 1.1–1.3. (You can compare your solutions to those in the back of the book.)

Problem C.

(a) Convert the expression $(x-1)(x+2)$ into a polynomial. That is, find the coefficients a, b, c such that

$$(x-1)(x+2) = ax^2 + bx + c.$$

(b) Find the roots of the polynomial $x^2 - 2x - 3$ and factor it as

$$x^2 - 2x - 3 = (x-\alpha)(x-\beta)$$

for suitable α and β.

Problem D. The *even* numbers are those of the form $2n$ for an integer n. The *odd* numbers are those of the form $2n+1$.

(a) Show that the sum of two odd numbers is even.

(b) Show that the product of two odd numbers is odd.

(c) Deduce from part (b) that if a product of two integers is even, then at least one of the two factors must be even.

Problem E. Consider the following two subsets of integers:

$$A = \{2, 3, 5, 7, 11, 13\} \qquad B = \{-3, -2, -1, 0, 1, 2, 3\}$$

(a) Give the cardinality of each set, that is $|A|$ and $|B|$.

(b) List all elements of their union $A \cup B$, and of their intersection $A \cap B$.

(c) Which of the statements below are true? Justify your answers.

(i) $7 \in A$

(ii) $0 \subseteq B$

(iii) $\{13\} \subseteq A$

(iv) $3 \in A \cap B$

(v) $A \not\subseteq B$

Problem F. Find the sum of all positive integers less than or equal to 1000:

$$T_{1000} = 1 + 2 + 3 + 4 + 5 + \cdots + 1000 =\!=?$$

Can you guess a formula for the sum $T_n = 1 + 2 + 3 + \cdots + n$, where n is any positive integer? Why do you think such a number is called *triangular*?

Problem G. Recall that if $a_1, a_2, \ldots, a_n$ is a list of numbers, we let $\sum_{i=1}^{n} a_i$ denote their sum $a_1 + a_2 + \cdots + a_n$. Evaluate the sums below.

(a) $\sum_{i=1}^{3} 2i - 1$

(b) $\sum_{i=1}^{4} i^2$

(c) $\sum_{i=1}^{5} i + i^3$