LECTURE 1

(Mon. JAN. 6, 2020)

A <u>set</u> X is a collection of <u>elements</u> x.

<u>Notation</u>: When x belongs to X we write $x \in X$.

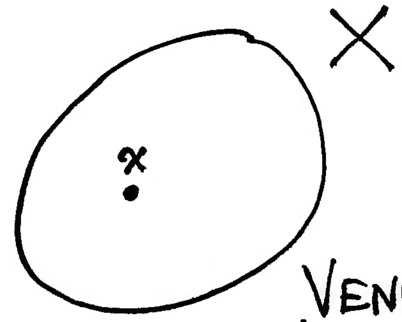
- Some standard sets:

$\mathbb{N} = \{1, 2, 3, \ldots\}$ natural numbers.

$\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$ all integers

$\mathbb{Q} = \{\text{fractions } \frac{a}{b}\}$ rational numbers.

$\mathbb{R} = \{\text{all decimal expansions}\}$ real numbers.



VENN diagram.

<u>EX</u> $\pi \in \mathbb{R}$ but $\pi \notin \mathbb{Q}$ ("π is irrational")

— FACT.

<u>Note</u>: The order doesn't matter. — for instance:

$$X = \{1, 2, 3\} = \{1, 3, 2\} = \{3, 1, 2\}$$

all describe the <u>same</u> set.

- Convention: There's an <u>empty</u> set $\emptyset$, thus

$x \in \emptyset$ is false for all x.

<u>EX</u>: $\{x \in \mathbb{N} : x < 0\} = \emptyset$.

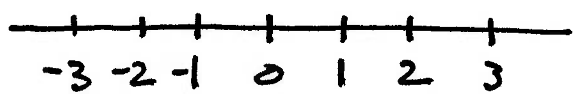
– Ways of describing a set:

EX. Let X be the collection of all even numbers $0, \pm 2, \pm 4, \ldots$ Can express X as

1) $X = \{2n : n \in \mathbb{Z}\}$ (parametric form)

2) $X = \{x \in \mathbb{Z} : x \text{ is a multiple of } 2\}$ conditional form.
(or "2 divides x")

EX $X = \{x \in \mathbb{Z} : |x| < \pi\} = \{0, \pm 1, \pm 2, \pm 3\}$



$= \{-3, -2, \ldots, 2, 3\}$.
(list form)

Def. When X is a finite set, its cardinality (or size) is the number of elements: $|X|$.

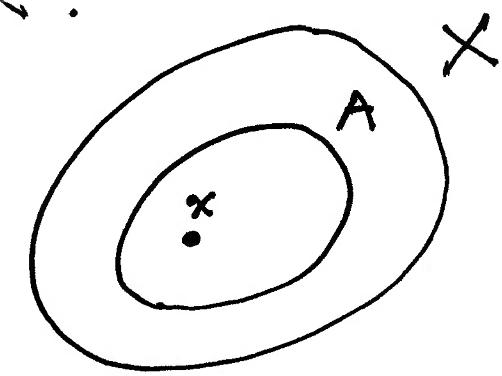
When X is infinite we'll write $|X| = \infty$.

EX $X = \{\text{students enrolled}\}$
$|X| = 60$.

A <u>subset</u> of X is a set A whose elements lie in X. That is,

$$x \in A \ \underline{\text{implies}} \ x \in X.$$

<u>Notation</u>: $A \subseteq X$.



<u>EX</u> ◦ Always have trivial subsets:

$$\emptyset \subseteq X \quad \text{and} \quad X \subseteq X.$$

◦ One-element subsets ("singletons"): $\{x\}$.

$$\{x\} \subseteq X \text{ is } \underline{\text{equivalent}} \text{ to } x \in X.$$

◦ $\mathbb{N} \subseteq \mathbb{Z}$, $\quad \mathbb{Z} \subseteq \mathbb{Q}$, $\quad \mathbb{Q} \subseteq \mathbb{R}$.

<u>Useful obs.</u>: Saying A is <u>not</u> a subset of X,

$$A \nsubseteq X$$

means there's <u>some</u> $x \in A$ which does <u>not</u> lie in X.

- For instance $\mathbb{Z} \nsubseteq \mathbb{N}$ because $0 \in \mathbb{Z}$ but $0 \notin \mathbb{N}$. (could've taken -1 or -100 ...)

- In general: If A and B are subsets of X,

$$A = B \quad \text{exactly when} \quad A \subseteq B \ \underline{\text{and}} \ B \subseteq A.$$

equality of sets.

<u>EX</u> $A = \{\pm 1\}$ and $B = \{x \in \mathbb{R} : x^2 = 1\}$
(both are subsets of $\mathbb{R}$)

i) $A \subseteq B$: $+1$ and -1 <u>are</u> solutions to eqn. $x^2 = 1$,
indeed $(-1)(-1) = 1$ etc.

ii) $B \subseteq A$: They're the <u>only</u> two solutions, since

$$x^2 - 1 = (x-1)(x+1) = 0$$

<u>implies</u> at least one of the two factors must be zero:

$$x - 1 = 0 \quad \text{or} \quad x + 1 = 0$$

Amounts to $x = 1$ or $x = -1$.

Conclude that $A = B$.