LECTURE 2

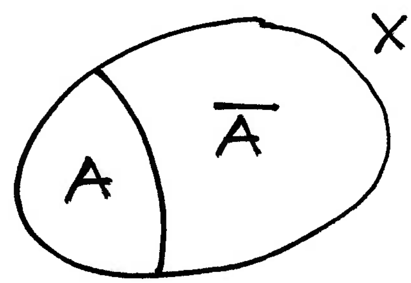
(Wed. JAN. 8, 2020)

__Def.__ If $A \subseteq X$ its __complement__ (in X) is the family of all elements of X __not__ in A:

$$\overline{A} = X - A = \{x \in X : x \notin A\}.$$

(sometimes A^c)

__Obs:__ $\overline{\overline{A}} = A$.



__Ex__ ($X = \mathbb{R}$)

$\overline{\mathbb{Q}} = \{x \in \mathbb{R} : x$ __not__ rational$\}$

"__irrational__ numbers". II. so $\pi \in \overline{\mathbb{Q}}$.

__Note:__
$$\begin{cases} \overline{\emptyset} = X \\ \overline{X} = \emptyset \end{cases}$$

__Def.__ The "__power set__" of X is the collection of all its subsets:

$$\mathcal{P}(X) = \{A : A \subseteq X\}.$$

(never __empty__)

- Thus, $\emptyset \in \mathcal{P}(X)$ and $X \in \mathcal{P}(X)$

__Ex__ $X = \{1,2,3\}$.

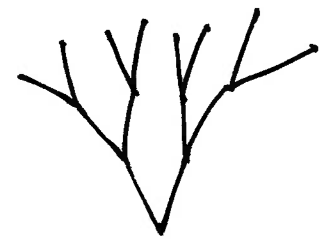
$$\mathcal{P}(\emptyset) = \{\emptyset\}.$$

$$\mathcal{P}(X) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, X\}.$$

— in general, if X is __finite__ with $|X| = n$, then $|\mathcal{P}(X)| = 2^n$ (exc.)

$X = \{x_1, x_2, \dots, x_n\}$, how many subsets? $A \subseteq X$

include x_1? yes/no. ... include x_n? yes/no.



Combinatorics: 2^n possibilities.

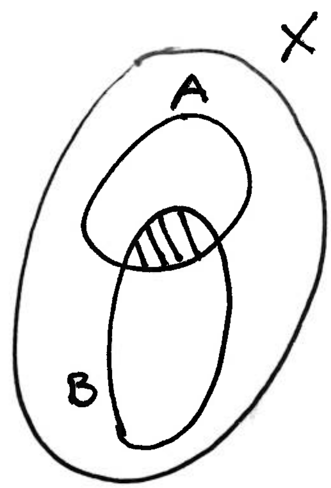
I.e., $|\mathcal{P}(X)| = 2^n$.

power set — consisting of all subsets $A \subseteq X$.

— Set operations: Fix a set X ("universal")

A and B two subsets of X.

<u>Def.</u> The intersection is



$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

The union is

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

possibly in both.

Ex. $A \cap \emptyset = \emptyset$

$A \cup \emptyset = A$

$A \cap X = A$

$A \cup X = X$.

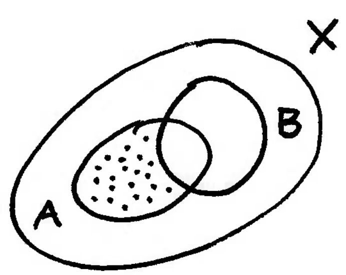
- Terminology: When $A \cap B = \emptyset$ — we say A, B "no overlap" are disjoint.

Ex ($X=\mathbb{Z}$) $A=\{x\in\mathbb{Z} : x\geqslant 3\}=\{3,4,5,\dots\}$

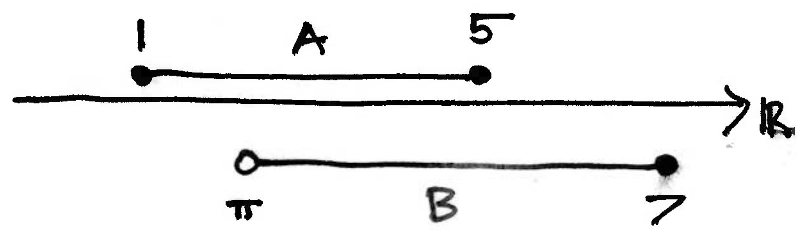
$B=\{x\in\mathbb{Z} : x\leqslant 5\}=\{\dots,3,4,5\}$.

Here, $A\cap B=\{3,4,5\}$ and $A\cup B=\mathbb{Z}$.

Ex $\left(\begin{array}{l} A\cup\overline{A}=X \\ A\cap\overline{A}=\varnothing . \end{array}\right)$.



Ex ($X=\mathbb{R}$) $A=[1,5]$
$B=(\pi,7]$



– Here $A\cap B=(\pi,5]$
$A\cup B=[1,7]$.

Def. The difference (or relative complement) is $A-B=\{x\in A : x\notin B\}$
$=A\cap\overline{B}$

- Useful observation: For any two $A,B\subseteq X$,

$$A\cup B=(A-B)\cup(B-A)\cup(A\cap B)$$

all mutually disjoint.

[Unions/intersections of three (or more) subsets are def. in the obvious way.] — more later..

p. 121.

<u>Theorem</u> (De Morgan's laws) $A, B \subseteq X$.

(i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

(ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

– <u>Why</u>? An $x \in X$ lies in the LHS precisely when
(i) x is <u>not</u> in $A \cup B$, i.e. when x is <u>not</u> in A, and <u>not</u> in B. This means $x \in \overline{A}$ and $x \in \overline{B}$; equivalently x lies in the RHS. ✓

– Can now <u>deduce</u> (ii) by applying (i) to $\overline{A}$ and $\overline{B}$:

$$\overline{\overline{A} \cup \overline{B}} = \overline{\overline{A}} \cap \overline{\overline{B}} = A \cap B.$$

using $\overline{\overline{A}} = A$ etc.

Taking the <u>complement</u>:

$$\overline{A} \cup \overline{B} = \overline{A \cap B}.$$

which is (ii). ✓