LECTURE 4

(Mon. JAN. 13, 2020)

– Let's check the 2^{nd} relation:

$$A \cap (B \cup C) \stackrel{?}{=} (A \cap B) \cup (A \cap C). \qquad (*)$$

$\subseteq$: x lies in the LHS precisely when $x \in A$ <u>and</u> $x \in B \cup C$. Two cases:

- If $x \in B$ then $x \in A \cap B$
- If $x \in C$ then $x \in A \cap C$

In either case x lies in $A \cap B$ or $A \cap C$, I.e., x lies in the RHS.

$\supseteq$: x lies in the RHS exactly when $x \in A \cap B$ or $x \in A \cap C$. Either way $x \in A$ and either $x \in B$ or $x \in C$. Thus $x \in A \cap (B \cup C)$.

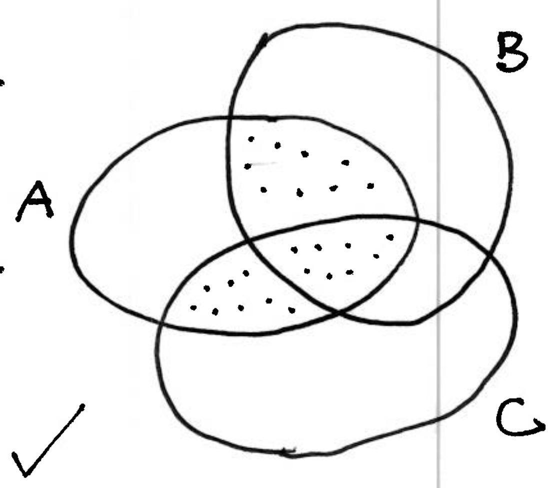
<u>Deduce</u> the 1^{st} relation by applying $(*)$ to $\overline{A}, \overline{B}, \overline{C}$ and <u>using</u> de Morgan:

$$\overline{\overline{A} \cap (\overline{B} \cup \overline{C})} = \overline{(\overline{A} \cap \overline{B}) \cup (\overline{A} \cap \overline{C})}$$

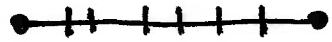
$$\overline{\overline{A}} \cup \overline{(\overline{B} \cup \overline{C})} \qquad \overline{(\overline{A} \cap \overline{B})} \cap \overline{(\overline{A} \cap \overline{C})}$$

$$A \cup (B \cap C) \qquad (A \cup B) \cap (A \cup C)$$

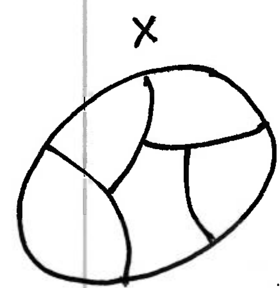
<u>VENN</u> <u>diagram</u>:



(integration: partition $[a,b]$)



Partitions



X set. "group its elements in boxes"

Ex ($X=\mathbb{Z}$) Every $x \in \mathbb{Z}$ is even or odd; not both!

$x = 2n$ (some $n \in \mathbb{Z}$) $\qquad x = 2n+1$

$E = \{2n : n \in \mathbb{Z}\}$

$F = \{2n+1 : n \in \mathbb{Z}\}$

Then $\mathbb{Z} = E \cup F$ and $E \cap F = \emptyset$. (disjoint)

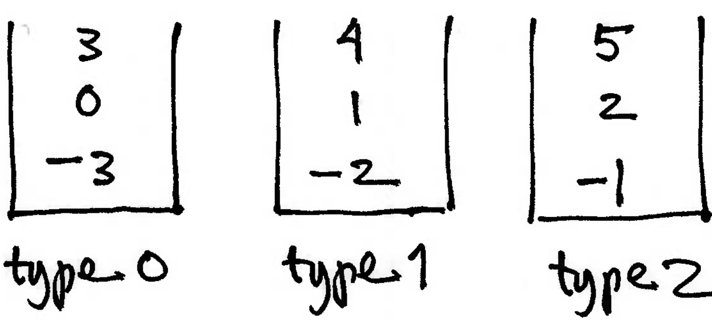
— another way to "archive" the integers:

now three boxes.

$A = \{3n : n \in \mathbb{Z}\}$ type 0.

$B = \{3n+1 : n \in \mathbb{Z}\}$ type 1.

$C = \{3n+2 : n \in \mathbb{Z}\}$ type 2.



2020 goes where?

$2020 = 3 \cdot 673 + 1$

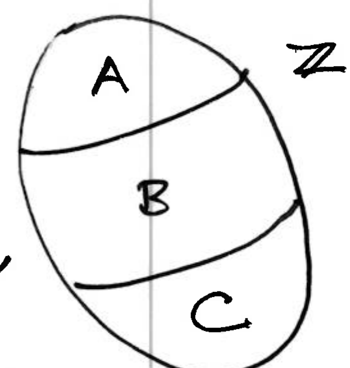
$-2020 = 3 \cdot (-674) + 2$.

Here

$\mathbb{Z} = A \cup B \cup C$

and A, B, C are pairwise disjoint:

$A \cap B = A \cap C = B \cap C = \emptyset$



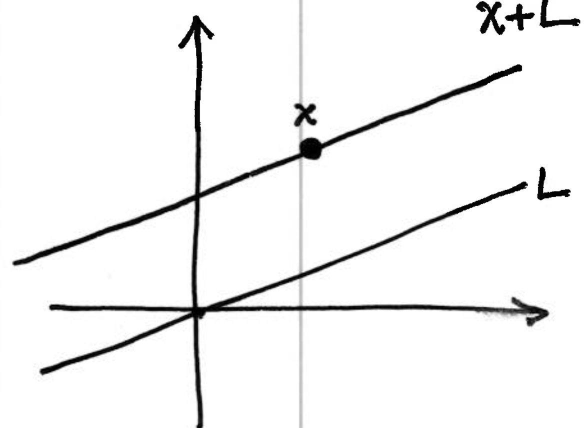
(stronger than saying $A \cap B \cap C = \emptyset$)

— In words, every integer lies in precisely one of the three subsets A, B, C.

We say $\{A, B, C\}$ is a partition of $\mathbb{Z}$.

Ex ($X = \mathbb{R}^2$ plane) Fix a line L through 0.

For $x \in X$ let $x + L$ be the line through x parallel to L.



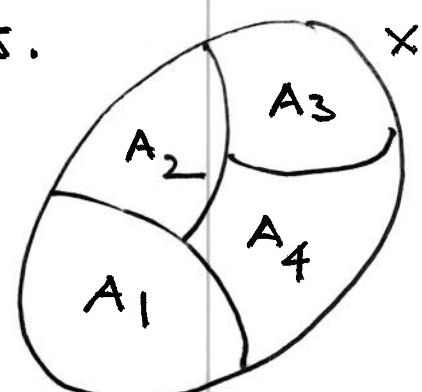
— the collection of all these lines gives a partition of $\mathbb{R}^2$:

- Every point $x \in \mathbb{R}^2$ lies on a unique line parallel to L.

Note: x, y may lie on the same line

— so $x + L = y + L$ although $x \neq y$. — may happen.

Def. A "partition" of X is a collection of non-empty subsets $A_\alpha \subseteq X$ indexed by $\alpha \in I$ s.t. every $x \in X$ belongs to exactly one of these subsets.



— more formally:

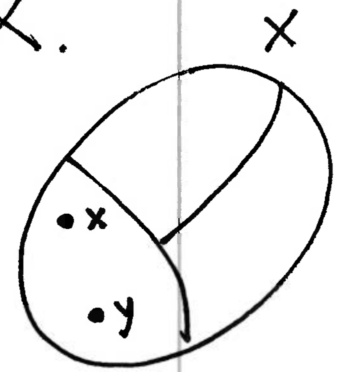
(1) $X = \bigcup_{\alpha \in I} A_\alpha$ and

(2) $A_\alpha \cap A_\beta \neq \emptyset$ then $A_\alpha = A_\beta$.

__Ex__ (cont) $I = \mathbb{R}^2$, and for $\alpha \in I$, $A_\alpha = \alpha + L$.

— Suppose we're given a <u>partition</u> of X.

We write $x \sim y$ when x and y lie in the <u>same</u> "chamber"



$x \sim y$.

- <u>Properties</u>:

  (i) $x \sim x$ (reflexive)

  (ii) $x \sim y$ is <u>equivalent</u> to $y \sim x$. symmetry

  (iii) $x \sim y$ and $y \sim z$ <u>implies</u> $x \sim z$ transitive.

(any three $x, y, z \in X$)

[later we'll say $\sim$ is an "<u>equivalence</u> <u>relation</u>" on the set X.]

.... and go back: From (i) — (iii) build a <u>partition</u> of X into "<u>equivalence</u> <u>classes</u>".