Math 109, Mathematical Reasoning, Midterm Exam Number 1

Monday, January 27th, 2020, 11-11:50am, APM B402A

- *Your Name*:
- *ID Number*:
- *Section*:

C01 (4:00 PM) C02 (5:00 PM)

Problem #	Points (out of 10)
1	
2	
3	
4	
5	
Total (out of 50):	

Problem 1. Let $X = \{1, 2, 3, \ldots, 10\}$ be the set consisting of all the positive integers less than or equal to 10.

(a) Give its cardinality $|X|$.

(b) How many subsets does X have?

(c) Let $A = \{2, 3, 5, 7\}$. List all the elements of its complement $\bar{A}$ in X.

Problem 2. Keep the set $X = \{1, 2, 3, \ldots, 10\}$ and the subset $A = \{2, 3, 5, 7\}$ introduced in Problem 1. Furthermore let $B = \{1, 3, 7, 9\}$.

(a) List all elements of their union $A \cup B$ and of their intersection $A \cap B$.

(b) List all elements of the two differences $A - B$ and $B - A$.

(c) Are A and B disjoint subsets?

Problem 3. Consider the sequence $A_1, A_2, A_3 \dots$ of intervals in $\mathbb{R}$ defined by $A_n = [0, \frac{1}{n}]$. (Both endpoints 0 and $\frac{1}{n}$ are included.)

(a) Which of the following statements are true, and which are false? Justify your answers.

(1) $A_{2020} \subseteq A_{2019}$

(2) $\frac{1}{2019} \in A_{2020}$

(b) Find their union $\bigcup_{n=1}^{\infty} A_n$ and intersection $\bigcap_{n=1}^{\infty} A_n$.

(c) Do the sets $A_n - A_{n+1}$ form a partition of $(0, 1]$ where $n = 1, 2, 3, \dots$?

Problem 4. For each of the mathematical statements below, indicate whether it is true or false. Justify your answers.

(a) $\forall a \in \mathbb{R} \;\; \exists b \in \mathbb{N} : a < b$

(b) $\exists a \in \mathbb{R} \;\; \forall b \in \mathbb{N} : a < b$

(c) $\forall n \in \mathbb{Z} : (n > 0) \vee (n < 0)$

(d) $\exists r \in \mathbb{Q} : (r < 0) \wedge (r > 0)$

(e) $x \in \mathbb{Q} \Longrightarrow x + \sqrt{2} \in \mathbb{Q}$

Problem 5. Recall that $\mathbb{Z}$ is partitioned into three subsets of numbers:

$$A = \{3n : n \in \mathbb{Z}\} \qquad B = \{3n + 1 : n \in \mathbb{Z}\} \qquad C = \{3n + 2 : n \in \mathbb{Z}\}.$$

(a) Which of these three subsets contains the number 1000?

(b) Verify the following two implications.

(i) $(x \in B) \wedge (y \in B) \Longrightarrow xy \in B$.

(ii) $(x \in C) \wedge (y \in C) \Longrightarrow xy \in B$.