Math 109, Mathematical Reasoning, Midterm Exam Number 2

Monday, February 24th, 2020, 11-11:50am, Peterson Hall 104

- *Your Name*:
- *ID Number*:
- *Section*:

C01 (4:00 PM) C02 (5:00 PM)

Problem #	Points (out of 10)
1	
2	
3	
4	
5	

Total (out of 50):

Problem 1. Define a relation $\sim$ on the set of real numbers $\mathbb{R}$ by declaring that

$$a \sim b \iff a - b \in \mathbb{Z}.$$

(Here $\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$ is the set of all integers.)

(a) Show that $\sim$ is an **equivalence** relation on $\mathbb{R}$.

(b) List all x in the range $0 < x < 3$ belonging to the equivalence class of $\sqrt{2}$.

Problem 2. In this problem [3] denotes the residue class of 3 modulo 7.

(a) Give the integer r in the range $0 \leq r < 7$ satisfying $r \equiv 100 (\mathrm{mod} 7)$.

(b) List all elements of [3] belonging to the open interval $(-20, 20)$.

(c) Give an $x \in [3]$ such that $x^2 \in [3]$ or disprove the existence of such an x.

Problem 3. Define a function $f : \{1, 2, 3, 4\} \longrightarrow \{1, 2, 3, 4\}$ as follows:

$$f(1) = 4 \qquad f(2) = 1 \qquad f(3) = 3 \qquad f(4) = 4.$$

(a) Is f surjective or injective? (Give a thorough explanation.)

(b) Calculate the value $(f \circ f)(2)$.

(c) List all elements of the two subsets $f(\{1, 2\})$ and $f^{-1}(\{3, 4\})$.

Problem 4.

(a) Suppose you are given 11 positive integers. Explain why at least two of them must have the same last digit.

(b) Suppose you are given 5 points in a disk of radius 1. Explain why at least two of them are within a distance $\sqrt{2}$ of each other.

Problem 5. Let $x_1, x_2, \ldots, x_n, \ldots$ be a sequence of real numbers converging to a number $x \in \mathbb{R}$.

(a) Prove that the sequence is bounded[1].

(b) Give an example of a bounded sequence which is not convergent.

[1]This means there is a constant $B > 0$ such that $|x_n| < B$ for all $n \in \mathbb{N}$.