Math 204A, Fall 2017

Algebraic Number Theory, HW 2

Due Wednesday October 18th 2017 at noon/after class

From [RME]:

- Section 3.1: *7* (page 29)
- Section 3.3: *3, 7* (pages 34–35)
- Section 4.2: *4* (page 45)

Problem A. Show that a UFD is a normal ring (i.e., integrally closed in its fraction field). Give an example of a non-normal ring.

Problem B. Let $d \in \mathbb{Z}$ be squarefree. Consider the quadratic extension $\mathbb{Q}(\sqrt{d})$.

(a) Find the minimal polynomial of $\frac{1+\sqrt{d}}{2}$ over $\mathbb{Q}$.

(b) Give a detailed proof that

$$\mathcal{O}_{\mathbb{Q}(\sqrt{d})} = \mathbb{Z}[\frac{1+\sqrt{d}}{2}]$$

if $d \equiv 1 \pmod 4$; and $\mathcal{O}_{\mathbb{Q}(\sqrt{d})} = \mathbb{Z}[\sqrt{d}]$ otherwise.

(c) Find an integral basis $\{\omega_1, \omega_2\}$ for $\mathcal{O}_{\mathbb{Q}(\sqrt{d})}$ and form the 2×2-matrix $A = \big(\mathrm{Tr}_{\mathbb{Q}(\sqrt{d})/\mathbb{Q}}(\omega_i\omega_j)\big)$. Compute $\det(A)$.