From [RME]:

- Section 3.1: 7 (page 29)
- Section 3.3: 3, 7 (pages 34–35)
- Section 4.2: 4 (page 45)

**Problem A.** Show that a UFD is a normal ring (i.e., integrally closed in its fraction field). Give an example of a non-normal ring.

**Problem B.** Let $d \in \mathbb{Z}$ be squarefree. Consider the quadratic extension $\mathbb{Q}(\sqrt{d})$.

(a) Find the minimal polynomial of $\frac{1+\sqrt{d}}{2}$ over $\mathbb{Q}$.

(b) Give a detailed proof that

$$
\mathcal{O}_{\mathbb{Q}(\sqrt{d})} = \mathbb{Z}\left[\frac{1+\sqrt{d}}{2}\right]
$$

if $d \equiv 1 \pmod{4}$; and $\mathcal{O}_{\mathbb{Q}(\sqrt{d})} = \mathbb{Z}[\sqrt{d}]$ otherwise.

(c) Find an integral basis $\{\omega_1, \omega_2\}$ for $\mathcal{O}_{\mathbb{Q}(\sqrt{d})}$ and form the $2 \times 2$-matrix $A = (\text{Tr}_{\mathbb{Q}(\sqrt{d})/\mathbb{Q}}(\omega_i \omega_j))$. Compute $\det(A)$.