From Neukirch’s book *Algebraic Number Theory*:

- Exercises:
  
  3 and 5 on page 115; 4 on page 123

**Problem A.** Let $K$ be a field with a non-archimedean absolute value $|·|$. 

(a) Let $x, y \in K$. Show that the strong triangle inequality

$$|x + y| \leq \max\{|x|, |y|\}$$

is an equality when $|x| \neq |y|$. 

(b) Let $x_1, \ldots, x_n \in K$. Show that

$$|x_1 + \cdots + x_n| = \max\{|x_1|, \ldots, |x_n|\}$$

provided the maximum on the right is achieved exactly once (that is some $|x_i|$ is larger than all $|x_j|$ for $j \neq i$). **Hint:** You may assume $i = 1$, in which case the assumption amounts to the inequality $|x_1| > \max\{|x_2|, \ldots, |x_n|\}$.

**Problem B.** Let $K$ be a field with a non-trivial non-archimedean absolute value $|·|$, and let $R = \{x \in K : |x| \leq 1\}$ be its valuation ring.

(a) Check that $R$ is integrally closed in its fraction field $\text{Frac}(R) = K$.

(b) Suppose $|K^\times|$ is discrete and choose a uniformizer $\pi \in R$. Explain why every nonzero ideal of $R$ is of the form $(\pi^i)$ for some $i \geq 0$. Deduce that $R$ is a Dedekind domain.