Math 204B, Winter 2019

Number Theory II, HW 2

Due Wednesday January 23rd in class (or by noon).

From Neukirch's book `Algebraic Number Theory`:

- Exercises:

  3 and 5 on page 115; 4 on page 123

Problem A. Let K be a field with a non-archimedean absolute value $|\cdot|$.

(a) Let $x, y \in K$. Show that the strong triangle inequality

$$|x+y| \leq \max\{|x|, |y|\}$$

is an equality when $|x| \neq |y|$.

(b) Let $x_1, \ldots, x_n \in K$. Show that

$$|x_1 + \cdots + x_n| = \max\{|x_1|, \ldots, |x_n|\}$$

provided the maximum on the right is achieved exactly once (that is some $|x_i|$ is larger than all $|x_j|$ for $j \neq i$). `Hint`: You may assume $i = 1$, in which case the assumption amounts to the inequality $|x_1| > \max\{|x_2|, \ldots, |x_n|\}$.

Problem B. Let K be a field with a non-trivial non-archimedean absolute value $|\cdot|$, and let $R = \{x \in K : |x| \leq 1\}$ be its valuation ring.

(a) Check that R is integrally closed in its fraction field $\mathrm{Frac}(R) = K$.

(b) Suppose $|K^\times|$ is discrete and choose a uniformizer $\pi \in R$. Explain why every nonzero ideal of R is of the form (π^i) for some $i \geq 0$. Deduce that R is a Dedekind domain.