MATH 31BH, WINTER 2017

HONORS MULTIVARIABLE CALCULUS, HW 1

Due Wednesday January 18th 2017 at 2:15 PM in Eric's box

Section 1.5 (p. 101): 1, 2, 3, 4, 5, 6, 7

Problem A. For $x \in \mathbb{R}^n$ and $r > 0$ let $B_r(x) = \{y \in \mathbb{R}^n : \|x - y\| < r\}$.

(a) Show that $B_r(x)$ is an open subset of $\mathbb{R}^n$.

(b) Prove that its closure is $\overline{B_r(x)} = \{y \in \mathbb{R}^n : \|x - y\| \leq r\}$.

Problem B. Use De Morgan's laws and the results of Exercise 1.5.3 to prove the following about closed subsets of $\mathbb{R}^n$.

(a) If $\{C_i\}_{i \in I}$ is any collection of closed sets, their intersection

$$\bigcap_{i \in I} C_i = \{x \in \mathbb{R}^n \text{ which lies in } \underline{\text{all}}\ C_i\}.$$

is also closed.

(b) If $\{C_i\}_{i \in I}$ is a *finite* collection of closed sets, their union

$$\bigcup_{i \in I} C_i = \{x \in \mathbb{R}^n \text{ which lies in } \underline{\text{some}}\ C_i\}.$$

is also closed.

(c) If $C_i = [0, 1 - \frac{1}{i}]$, describe the union $\bigcup_{i=1}^{\infty} C_i$. Is it an interval?