Math 31BH, Winter 2017

Honors Multivariable Calculus, HW 2

Due Monday January 23rd 2017 at 2:15 PM in Eric's box

Section 1.5 (p. 101): 9, 13, 14, 16, 17 (part 1 only), 21, 22(a)

Problem A. For two vectors $x, y \in \mathbb{R}^n$ let $D(x, y) = \max_{i=1,\dots,n} |x_i - y_i|$.

(a) Show that D defines a metric on $\mathbb{R}^n$.

(b) Give a geometric description of the open balls defined by D, i.e.

$$B_r^D(x) = \{y \in \mathbb{R}^n : D(x, y) < r\}.$$

Draw $B_3^D((2, 1))$ in the plane.

(c) For any two $x, y \in \mathbb{R}^n$ show the inequalities

$$D(x, y) \leq d(x, y) \leq \sqrt{n} \cdot D(x, y),$$

where $d(x, y) = \|x - y\| = \sqrt{\sum_{i=1}^n (x_i - y_i)^2}$ is the usual metric on $\mathbb{R}^n$.

Problem B. Find the interior, closure, and boundary of the sets below.

(a) The set of all integers $\mathbb{Z} = \{0, \pm 1, \pm 2, \dots\}$ in $\mathbb{R}$.

(b) The "half-open" unit square $[0, 1) \times (0, 1]$ in $\mathbb{R}^2$.

(c) The subset $\{(\frac{1}{a}, \frac{1}{b}) : a, b \in \mathbb{N}\}$ of $\mathbb{R}^2$.

(`Hint`: Draw them.)