Math 31BH, Winter 2017

Honors Multivariable Calculus, HW 6

Due Wednesday February 22nd 2017 at 2:15 PM in Eric's box

Section 1.8 (p. 143): 4, 7, 8, 9, 10[1], 11, 12.

Problem A. We express a function $f : \mathbb{R}^2 \to \mathbb{R}$ in *polar* coordinates as follows:

$$\tilde{f}(r, \theta) := f(r\cos\theta, r\sin\theta).$$

(a) Assuming f is differentiable explain why $\tilde{f}$ is differentiable.

(b) Under the assumption in (a) verify that

(i) $r\frac{\partial \tilde{f}}{\partial r}(r, \theta) = x\frac{\partial f}{\partial x}(x, y) + y\frac{\partial f}{\partial y}(x, y)$.

(ii) $\frac{\partial \tilde{f}}{\partial \theta}(r, \theta) = -y\frac{\partial f}{\partial x}(x, y) + x\frac{\partial f}{\partial y}(x, y)$.

Here $x = x(r, \theta) = r\cos\theta$ and $y = y(r, \theta) = r\sin\theta$.

Problem B. Define a function $f : \mathbb{R}^2 \to \mathbb{R}$ by the formula $f(x, y) = \frac{xy(x^2-y^2)}{x^2+y^2}$ when $(x, y) \neq (0, 0)$, and declare that $f(0, 0) = 0$.

(a) Calculate the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at <u>all</u> points (x, y).

(b) Evaluate the partial derivatives on the two axes $x = 0$ and $y = 0$; i.e., find

$$\frac{\partial f}{\partial x}(0, y), \qquad \frac{\partial f}{\partial y}(0, y), \qquad \frac{\partial f}{\partial x}(x, 0), \qquad \frac{\partial f}{\partial y}(x, 0).$$

(c) Compute the following second order partial derivatives at the origin;

$$\frac{\partial^2 f}{\partial x^2}(0, 0), \qquad \frac{\partial^2 f}{\partial x \partial y}(0, 0), \qquad \frac{\partial^2 f}{\partial y \partial x}(0, 0), \qquad \frac{\partial^2 f}{\partial y^2}(0, 0).$$

(d) Conclude that f is of class $\mathcal{C}^1$ but <u>not</u> $\mathcal{C}^2$. (cf. Def. 1.9.7 on p. 149.)

[1]`Hint`: Use Problem A part (b)(ii) on this problem set for part (b) of exercise 1.8.10.