Math 31BH, Winter 2017

Honors Multivariable Calculus, HW 7

Due Monday February 27th 2017 at 2:15 PM in Eric's box

Section 1.8 (p. 143): 2

Section 3.2 (p. 312): 1(a), 2, 3 – only the tangent *line* questions.

Section 3.3 (p. 323): 1, 2, 9

Problem A. Let $f : U \to \mathbb{R}$ be a differentiable function defined on an open set $U \subset \mathbb{R}^n$. Fix $a \in U$ and let L be the *level set* containing the point a, i.e.

$$L = \{x \in U : f(x) = f(a)\}.$$

Suppose $\alpha : (-\epsilon, \epsilon) \to \mathbb{R}^n$ is a differentiable function mapping into L, with $\alpha(0) = a$. Show that $\alpha'(0)$ is *orthogonal* to $\nabla f(a)$ – in other words that

$$\nabla f(a) \bullet \alpha'(0) = 0.$$

Problem B. Let $f : U \to \mathbb{R}$ be a differentiable function defined on an open set $U \subset \mathbb{R}^n$. Consider its *graph* $\Gamma \subset U \times \mathbb{R}$, i.e.

$$\Gamma = \{(x, f(x)) : x \in U\}.$$

Suppose $\beta : (-\epsilon, \epsilon) \to \mathbb{R}^{n+1}$ is a differentiable function mapping into Γ, with $\beta(0) = (a, f(a))$. Show that $\beta'(0)$ lies on the graph of $T(x) = \nabla f(a) \bullet x$ – in other words that

$$\beta'(0) = (x, T(x))$$

for some $x \in \mathbb{R}^n$.