1. Compact subsets of metric spaces are closed (by a theorem in Rudin) so $K$ is closed. Finite unions of closed sets are again closed (by a theorem in Rudin) so $F \cup K$ is closed.

2. (a) $K$ is compact if and only if every open cover of $K$ possesses a finite subcover. By open cover we mean a collection $\{V_\alpha\}_{\alpha \in A}$ of open sets such that $K \subset \bigcup_{\alpha \in A} V_\alpha$. By a finite subcover, we mean any finite subcollection $\{V_{\alpha_1}, \ldots, V_{\alpha_n}\}$ such that $\bigcup_{n=1}^n V_{\alpha_j}$. (b) Suppose that $K_1 \cup K_2 \subset \bigcup_{\alpha \in A} V_\alpha$. Then in particular, $K_1 \subset \bigcup_{\alpha \in A} V_\alpha$ and $K_2 \subset \bigcup_{\alpha \in A} V_\alpha$. Since each of $K_1$ and $K_2$ are compact, there are $\alpha_1, \ldots, \alpha_n \in A$ and $\beta_1, \ldots, \beta_m \in A$ such that $K_1 \subset \bigcup_{j=1}^n V_{\alpha_j}$ and $K_2 \subset \bigcup_{j=1}^m V_{\beta_j}$. It follows then that $(K_1 \cup K_2) \subset \bigcup_{n=1}^n V_{\alpha_j} \cup \bigcup_{m=1}^m V_{\beta_j}$ so $K_1 \cup K_2$ is compact.

(c) Define $V_n = (1 + \frac{1}{n}, 3)$ for $n \geq 2$. Then $(1, 2] \subset \bigcup_{n=1}^\infty V_n$, but there is no possible finite subcover: suppose that $V_{n_1}, \ldots, V_{n_k}$ covered $(1, 2]$. Choose $N = \max\{n_1, \ldots, n_k\}$, and choose $x \in (1, 1 + \frac{1}{N})$. Then $x \in (1, 2]$ but $x$ is in no $V_{n_j}$, a contradiction.

3. We must show that every point of $(E')^c$ is an interior point. So choose $p \in (E')^c$. Since $p$ is not a limit point of $E$, there is a neighborhood $N_r(p)$ that does not intersect $E$, except perhaps in $p$ alone. We claim that $N_r(p) \subset (E')^c$: choose any $q \in N_r(p)$. Choose $s < \min\{d(p, q), r - d(p, q)\}$. Then $p \notin N_s(q)$ and $N_s(q) \subset N_r(p)$, so $N_s(q) \cap E = \emptyset$. Thus $q \in (E')^c$, so we are done.

4. For $n \geq 0$, we will denote by $P_n(S)$ the set of all polynomials of degree $n$ with coefficients in $S$. $P_0(S)$ is in bijection with $S$ and is thus countable, and for $n \geq 1$, $P_n(S)$ is in bijection with the set of $(n+1)$-tuples $S_n := \{(a_0, a_1, \ldots, a_n) \mid a_i \in S\}$.

For any $n \geq 1$, $S_n$ is countable by a theorem in Rudin, so each $P_n(S)$ is countable. It is clear that $P(S) = \bigcup_{n=0}^\infty P_n(S)$, and since a countable union of countable sets is again countable (by a theorem in Rudin), $P(S)$ is countable.