2.6

3. $\nabla f = (yz, xz, xy)$

$\nabla f(1,0,1) = (0,1,0)$

$\nabla f \cdot u = (0,1,0) \cdot (1,0,-1) = 0.$

4. $f = x^2 + 2y^2 + 3xz$; $\nabla f = (2x+3z, 4y, 3x)$

$\nabla f(1,2,1/3) = (3,8,3)$

$\nabla f(1,2,1/3) \cdot (x-1, y-2, z-1/3) = 0$

$\cancel{(x-1) + 2(y-2) +}$

$= 3(x-1) + 8(y-2) + 3(z-1/3)$

$= 3x + 8y + 3z - 20$

plane is $20 = 3x + 8y + 3z.$

(6a) $\nabla f = \left(-\frac{1}{2}\cdot(2x)\cdot(x^2+y^2+z^2)^{-3/2}, -\frac{1}{2}(2y)(x^2+y^2+z^2)^{-3/2},\right.$
$\left.-\frac{1}{2}(2z)(x^2+y^2+z^2)^{-3/2}\right)$
$= (-x, -y, -z) / (x^2+y^2+z^2)^{3/2}$.

(9) Let $f = \cos xy - e^z + 2$

$\nabla f = (-y \sin xy, -x \sin xy, -e^z)$

$\nabla f(1, \pi, 0) = (0, 0, -1)$.

So $\vec{n} = (0, 0, -1)$ (or $\vec{n} = (0,0,1)$ if we use $f = e^z - 2 - \cos xy$ at beginning — both are correct).

(12) Let $f(x,y) = z$. Then $f = (1-x^2-y^2)^{1/2}$ is the same as $x^2+y^2+z^2 = 1$. Using p167 the tangent plane is $\nabla(x^2+y^2+z^2)(x_0,y_0,z_0)\cdot(x-x_0, y-y_0, z-z_0) = 0$ or

$x_0(x-x_0) + y_0(y-y_0) + z_0(z-z_0) = 0$ where $z_0 = f(x_0,y_0)$.

By p51 this plane's normal vector is $(x_0, y_0, z_0) = (x_0, y_0, f(x_0,y_0))$

(24 b) Let $g = f \circ c : \mathbb{R} \to \mathbb{R}$. Want to find t s.t. $g'(t) = 0$ (like 20 A).

$$g(t) = (\cos t)^2 + (2 \sin t)^2 = 1 + 3 \sin^2 t$$

$$g'(t) = 6 \sin t \cos t$$

$= 0$ when $\sin t = 0$ or $\cos t = 0$

or when $t = 0, \pi, 2\pi$ or $t = \pi/2, 3\pi/2$

$$g''(t) = 6 \cos^2 t - 6 \sin^2 t = 6 \cos 2t$$

t	g''	min/max	$g(t)$
0	> 0	min	1
$\pi/2$	< 0	max	4
π	> 0	min	1
$3\pi/2$	< 0	max	4
2π	> 0	min	1

So maxima at $t = \pi/2, 3\pi/2$

minima at $t = 0, \pi, 2\pi$.

3.2

(1) $\cancel{f((0,0)+\underline{h}) = f(0,0) + \sum_{i=1}^{2} h_i \frac{\partial f}{\partial x}}$

(2) $\cancel{f((0,0)+\underline{h})}$

(2) $$f((0,0)+(h_x,h_y)) = f(0,0) + h_x \frac{\partial f}{\partial x}(0,0) + h_y \frac{\partial f}{\partial y}(0,0)$$
$$+ \frac{1}{2} h_x^2 \frac{\partial^2 f}{\partial x^2}(0,0) + \frac{1}{2} h_y^2 \frac{\partial^2 f}{\partial y^2}(0,0) + h_x h_y \frac{\partial^2 f}{\partial x \partial y}(0,0)$$

$f(0,0) = 1$, $\frac{\partial f}{\partial x} = \frac{-2x}{(x^2+y^2+1)^2} = 0$ $\frac{\partial f}{\partial y} = \frac{-2y}{(x^2+y^2+1)^2} = 0$

$$\frac{\partial^2 f}{\partial x^2} = \frac{-2(x^2+y^2+1)^2 + (2x)\cdot 2(x^2+y^2+1)\cdot 2x}{(x^2+y^2+1)^4} = -2$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{-2(x^2+y^2+1)^2 + (2y)\,2\cdot(x^2+y^2+1)2y}{(x^2+y^2+1)^4} = -2$$

$$\frac{\partial^2 f}{\partial x \partial y} = (-2x)\cdot(-2)(2y)\cdot(x^2+y^2+1)^{-3} = 0$$

So $f(h_x,h_y) = 1 - h_x^2 - h_y^2$

(6) $f(1,0) = 1$; $\frac{\partial f}{\partial x} = 2(x-1)e^{(x-1)^2}\cos y = 0$

$\frac{\partial f}{\partial y} = -e^{(x-1)^2}\sin y = 0$ $\qquad \frac{\partial^2 f}{\partial x^2} = 2e^{(x-1)^2}\cos y + 4(x-1)^2 e^{(x-1)^2}\cos y$

$= 2$

$\frac{\partial^2 f}{\partial y^2} = -e^{(x-1)^2}\cos y = -1$ $\qquad \frac{\partial^2 f}{\partial x \partial y} = -2(x-1)e^{(x-1)^2}\sin y = 0.$

so $f((1,0)+(h_x,h_y)) = \cancel{1 + h_x\cdot 0 + h_y \cdot 0 + \frac{1}{2}\left[h_x^2\cdot 2 + 2h_xh_y\cdot 0 + h_y^2\cdot(-1)\right]}$

$$= 1 + (h_x-1)\cdot 0 + h_y\cdot 0 + \frac{1}{2}\left[(h_x-1)^2\cdot 2 + 2(h_x-1)\cdot h_y\cdot 0 + h_y^2\cdot(-1)\right]$$

$$= 1 - (h_x-1)^2 - h_y^2/2$$