(3) Let S_1 = upper hemisphere. Then

$$\iint_{S_1} E \cdot dS = \iint_{S_1} (E \cdot n)\, dS$$

$$= 2\iint_{S_1} (x^2+y^2+z^2)\, dS$$

$$= 2\iint_{S_1} dS = 2\, A(S)/2 = 4\pi.$$

Let S_2 = base. Then $\vec{n} = (0,0,-1)$

Since S_2 parametrized by $(x,y,0)$ have

$$E \cdot n = 0 \Rightarrow \iint_{S_2} E \cdot n\, dS = 0.$$

$$\Rightarrow \iint_S E \cdot dS = 4\pi.$$

(2) flux is $\iint_S -k\nabla T \cdot dS = \iint_S (-k,0,0)\cdot n \, dS$

where $n = (x, y, z)$

so have $\iint_S -kx \, dS$ let

$T(\theta, \phi) = (\cos\theta \sin\phi, \sin\theta \sin\phi, \cos\phi)$ then

we have $\iint_S f \, dS = \iint_D f(T) \| T_\phi \times T_\theta \|$

$$= \int_0^{\pi}\int_0^{2\pi} -k \cos\theta \sin\phi \cdot \sin^2\phi \, d\theta \, d\phi$$

as $\| T_\phi \times T_\theta \| = \sin^2\phi$.

$= 0.$

(5) $\nabla \times F = (2x^3yz, -3x^2y^2z, \cancel{\ }\,{-2})$

let $g(x,y) = z = -\sqrt{\frac{1-x^2-y^2}{3}}$

$$\vec{n} = \left(\frac{-x}{\sqrt{3(1-x^2-y^2)}}, \frac{-y}{\sqrt{3(1-x^2-y^2)}}, 1\right) = \left(\frac{-x}{z}, \frac{-y}{z}, 1\right)$$

So $$\iint_S \nabla \times F \cdot dS = \iint (\nabla \times F \cdot n)\, dS$$

$$= \iint_{x^2+y^2 \le 1} (2x^3yz, -3x^2y^2z, -2) \cdot \left(\frac{-x}{z}, \frac{-y}{z}, 1\right)$$

$$= \iint_{x^2+y^2 \le 1} -2x^4y - 3x^2y^3 - 2 \, dx\, dy$$

Then use polar coords ...

$$= \int_0^{2\pi}\int_0^1 (-2 - 2r^5\cos^4\theta \sin\theta - 3r^5\cos^2\theta \sin^3\theta)\, r \, dr\, d\theta$$

7. $\iint_S F \cdot dS = \iint_S (F \cdot n)\, dS$

$$= \iint_S (x+3y^5,\ y+10xz,\ z-xy) \cdot (x,y,z)\, dS$$

$$= \iint_S x^2+3xy^5+y^2+10xyz+z^2-xyz\ dS$$

Break $S = S_1 \cup S_2$ $\quad$ S_1 = upper hemisphere

S_2 = base $\quad x^2+y^2 \le 1,\ z=0$

On S_1: $x^2+y^2+z^2=1$ so

$\iint_{S_1} F \cdot n\, dS = \iint_{S_1} 1+3xy^5+9xyz\ dS$, use spherical coords.

On S_2: $z=0$ so $\iint_{S_2} F\cdot n\, dS = \iint_{S_2} x^2+y^2+3xy^5$,

use polar coords

10. $n = (x, y, 0)$. So

$$\iint_S F \cdot n \, dS = \iint_S x + y \, dS$$

Let $\Phi(z, \theta) = (\cos\theta, \sin\theta, z)$ so $\|\Phi_z \times \Phi_\theta\| =$

$$\left| \begin{matrix} i & j & k \\ 0 & 0 & 1 \\ -\sin\theta & \cos\theta & 0 \end{matrix} \right| = \|(\cos\theta, -\sin\theta, 0)\| = 1.$$

$$= \iint_S x + y \, dS = \int_0^{2\pi} \int_0^1 (\cos\theta + \sin\theta) \cdot 1 \, dz \, d\theta.$$

(18.)

a) $\iint_S F \cdot n \, dS = \iint_S (x,y,0) \cdot (x,y,z) \, dS$

$= \iint_S x^2+y^2 \, dS$

$= \int_0^{\pi/2} \int_0^{2\pi} \left((\cos\theta \sin\phi)^2 + (\sin\theta \sin\phi)^2 \right) \sin^2\phi \, d\theta \, d\phi$

b) Same logic

c) Rote calculation.

8.1 ① $\int y\,dx - x\,dy = \int_{-1}^{1}\int_{-1}^{1} -1-1 = \int_{-1}^{1}\int_{-1}^{1} -2$

$= (-2)\cdot(2)\cdot(2)$

$= -8.$

③ Show $\int 2y\,dx + x\,dy = \iint_D 1-2\,dS.$

$\int 2y\,dx + x\,dy = \int_0^{2\pi} 2(\sin\theta)(-\sin\theta) + (\cos\theta)(\cos\theta)\,d\theta$

$= \int_0^{2\pi} \cos^2\theta - 2\sin^2\theta$

$= -\pi$

$\iint_D -1\,dS = -1\cdot\pi$

(8) $\int F\cdot n \, ds = \iint_D \operatorname{div} F = 0$

Directly: WTS $\int_{\partial D} F\cdot n \, ds = 0$

''

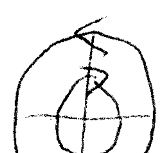
$\int_{\partial D} (y,-x)\cdot(x,y) = 0.$

(9) Show $\iint_{\text{annulus}} \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \int_{\partial \text{ annulus}} P\,dx + Q\,dy.$

∂ annulus $= \partial S_1 \cup \partial S_2$ where ∂S_1 is circle of radius $\sqrt{9}$, ∂S_2 circle of radius $\sqrt{5}$.

$$\iint \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \int\int 3x^2 + 3y^2$$

$$= \int_0^{2\pi}\int_{\sqrt{9}}^{\sqrt{5}} 3r^2\cdot r\,dr\,d\theta$$



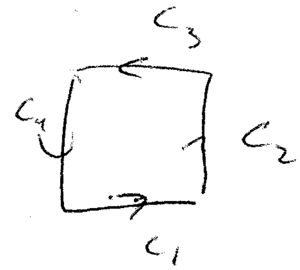
$\int_{\partial \text{ann.}} P\,dx + Q\,dy = -\int_{\partial S_1} + \int_{\partial S_2}$ ∂S_1 has negative orientation so negative sign.

$= -\int_{\partial S_1}^{2\pi} 9^2(2\cos^3\theta - \sin^3\theta)\cdot(-\sin\theta) + 9^2(\cos^3\theta + \sin^3\theta)\,d\theta + \int_{\partial S_2} \cdots$

(12) Because P, Q are not C^1 functions.

e.g. Their derivatives aren't continuous.

(13) Break into 4 pieces



& do line integrals on each

e.g. ~~C_1 param~~ C_1 parametrized by $(t,0)$

C_2 " $(1,t)$

C_3 " $(1-t,1)$

C_4 " $(0,1-t)$

Then solve $\displaystyle\int_{C_t} = \int_{C_1} + \int_{C_2} + \int_{C_3} + \int_{C_4}$

e.g.
$$\int_{C_1} = \int_0^1 (0^2+t^3)\cdot 1\,dt + t^4\cdot 0\,dt$$
$$= \int_0^1 t^3\,dt.$$

etc.

(14) Routine.

(20) Let $P = -y$ $Q = 0$. Then Greens Thm says

$\int -y\,dx = A(D)$. Let $P = 0$, $Q = x$.

Greens says $\int x\,dy = A(D)$.

8.2

(3) Show $\iint_S \nabla \times F \cdot dS = \int_{\partial S} F \cdot ds$ parametrize by $(\cos\theta, \sin\theta, 0)$

$\qquad\qquad /\!/ \qquad\qquad\qquad\qquad\qquad\qquad \|$

$\iint 0 = 0$. $\qquad \int_0^\pi (\cos\theta, \sin\theta, 0) \cdot c'(t)\,dt$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \shortparallel$

$\int_0^\pi (-\sin\theta\cos\theta + \sin\theta\cos\theta) = 0$

(6) $\nabla \times F = 0$ so $\iint_S \nabla \times F = 0$. WTS $\int_{\partial S} F \cdot ds = 0$.

$\partial S = S_1 \cup S_2 \cup S_3$ where
$S_1 = (1-t)(1,0,0) + t(0,1,0)$
$S_2 = (1-t)(0,1,0) + t(0,0,1)$
$S_3 = (1-t)(0,0,1) + t(1,0,0)$

integrate each — ex. $\int_{S_1} F \cdot ds = \int_0^1 (0,0,(1-t)t) \cdot (-1,1,0) = 0$.

(10) $\partial S = \emptyset$ so $\iint_S \nabla \times F \, dS = \int_{\partial S} F \cdot ds$

$= \int_{\emptyset} F \cdot ds = 0.$

(23) a) Since $\int_S F \cdot dS = \int F \cdot n \, dS$

$= \int (x^2, \overset{2xy+x}{\cancel{2xy+x}}, z) \cdot (0,0,1)$

$= \int z \, dS$

parametrize by $(r\cos\theta, r\sin\theta, 0)$

$= \int 0 = 0.$

b) $\int_C F \cdot ds = \int_0^{2\pi} \underbrace{(\cos^2\theta, 2\cos\theta\sin\theta + \cos\theta, 0)}_{F(c(t))} \cdot \underbrace{(-\sin\theta, \cos\theta, 0)}_{c'(t)}$

$= \int_0^{2\pi} -\sin\theta\cos^2\theta + 2\cos^2\theta\sin\theta + \cos^2\theta \, d\theta.$

c) $\nabla \times F = \cancel{} (0,0,1+2y)$ Show

$\iint_S \nabla \times F \, dS = \int_C F \cdot ds$ (from (b))

$\| $

$\iint_S (0,0,1+2y) \cdot (0,0,1) \, dS = \iint_S 1+2y \, dS$

8.3 (4) Note $F = \nabla(x^2yz - \cos x)$ so

$$\int_c F \cdot ds = f(c(\pi)) - f(c(0)).$$

(7) No since $\nabla \times F = (0, 0, -x) \neq 0$

(10) $\int_c F \cdot ds$ parametrize by $(\cos\theta, \sin\theta, 0)$ ~~$(r\cos\theta, r\sin\theta, \theta)$~~

$=$

~~$\int_0^{2\pi} (r^2 \cos\theta \sin\theta, 0, 0) \cdot (-\sin\theta, \cos\theta, 0)$~~

$= \int_0^{2\pi} (\cos\theta \sin\theta, 0, 0) \cdot (-\sin\theta, \cos\theta, 0) = \int_0^{2\pi} -\sin^2\theta \cos\theta$

$= 0.$

(12) a) $\displaystyle\int \frac{-y\,dx + x\,dy}{x^2+y^2} = \int_0^{2\pi} \frac{(-\sin\theta)(-\sin\theta) + \cos\theta\cos\theta}{1} = \int_0^{2\pi} 1 = 2\pi$

b) Since $\int_c F \cdot ds \neq 0$ is not conservative.

as c is closed

c) No. F is not a C^1 function.

13 a) $f = \frac{x^2}{2} + \frac{y^2}{2}$

c) $f = \frac{x^3}{3} + xy^2$

20 Use formula in Ex 16.

24 a) $\nabla \times G = 0 \Rightarrow$ irrotational

b) $\int_C G \cdot ds \neq 0$ if C = unit circle in x-y plane

$\Rightarrow$ there is circulation

c) F is C^1 so thms hold for it, G is not C^1 so thms don't hold.

8.4

(2) $\iint_S F \cdot ds = \iiint_U \operatorname{div} F$

$= \iiint_U x^2+y^2+z^2$

spherical coords.

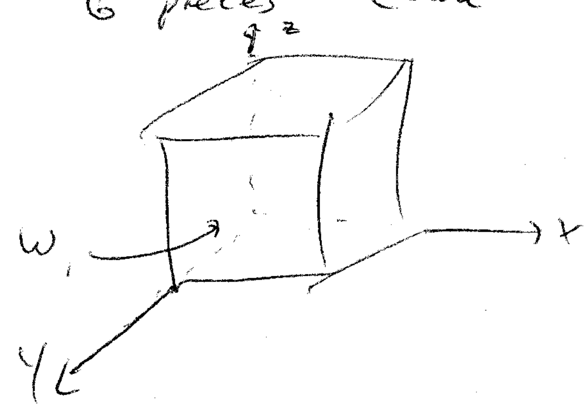
$= \int_0^{\pi}\int_0^{2\pi}\int_0^1 \rho^2 \cdot \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

(3) $\iint_{\partial W} F \cdot ds = \iiint_W \operatorname{div} F \, dV = \iiint_W 3 \, dV$

$= 3 \cdot V(\text{unit cube})$

$= 3.$

For $\iint_{\partial W} F \cdot dS$, break ∂W into 6 pieces (each face of the cube) & integrate each



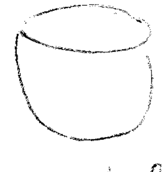
Ex let w_1 be the front:

w_1 parametrized by $(x,1,z)$ with $\vec{n} = (0,1,0)$

so $\iint_{w_1} F \cdot ds = \iint_{w_1} F \cdot n \, ds = \iint_{w_1} 1 \, ds = 1.$

Repeat for $w_2 - w_6$

(5a) $x^2+y^2 = z$ is a paraboloid



So $x^2+y^2 \le z$ is stuff under paraboloid.

$$\iint_{\partial W} F\cdot dS = \iiint_W \operatorname{div} F\, dV = \iiint_W x\, dV$$

$$= \int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\int_0^{x^2+y^2} x\, dz\, dx\, dy$$

$$= \int_0^{2\pi}\int_0^{1}\int_0^{r^2} r\cos\theta \cdot r\, dz\, dr\, d\theta$$

(8) $$\iint_S F\cdot dS = \iiint_{\text{Ball}} \operatorname{div} F\, dV = 3\iiint x^2+y^2+z^2\, dx\, dy\, dz$$

$$= 3\int_0^{\pi}\int_0^{2\pi}\int_0^{1} \rho^2 \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$$