Useful Formulas

1. $\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
2. $\displaystyle\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
3. (First Fund. Theorem of Calc.) $\displaystyle\int_a^b f(x)\,dx = F(b) - F(a)$, where $F'(x) = f(x)$.
4. Average value of $f(x)$ on $[a,b]$ $= \frac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$
5. If $m \le f(x) \le M$ on $[a,b]$, then $m(b-a) \le \displaystyle\int_a^b f(x)\,dx \le M(b-a)$
6. If $f(x) \le g(x)$ on $[a,b]$, then $\displaystyle\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$
7. (Second Fund. Theorem of Calc.) $\displaystyle\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$
8. $\displaystyle\int f\,dg = f\,g - \int g\,df$
9. $\sin^2 x + \cos^2 x = 1$
10. $\tan^2 x + 1 = \sec^2 x$
11. (a) If $0 \le f(x) \le g(x)$ and $\displaystyle\int_a^\infty g(x)\,dx$ converges, then $\displaystyle\int_a^\infty f(x)\,dx$ also converges.

    (b) If $0 \le g(x) \le f(x)$ and $\displaystyle\int_a^\infty g(x)\,dx$ diverges, then $\displaystyle\int_a^\infty f(x)\,dx$ also diverges.
12. (a) $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ converges for $p > 1$ and diverges for $p \le 1$.

    (b) $\displaystyle\int_0^\infty e^{-ax}\,dx$ converges for $a > 0$ and diverges for $a \le 0$.
13. Volume of Revolution$= \pi\displaystyle\int_a^b [(\text{outer radius})^2 - (\text{inner radius})^2]\,dx$