

Name: _____ PID: _____

TA: _____ Sec. No: _____ Sec. Time: _____

Math 10B.
Final Examination
March 21, 2006

Read each question carefully, and answer each question completely.

Show all of your work. No credit will be given for unsupported answers.

Write your solutions clearly and legibly. No credit will be given for illegible solutions.

1. (4 points) If $f(x)$ is even and $\int_{-3}^3 \{f(x) - 2\} dx = 6$, find $\int_0^3 f(x) dx$.

#	Points	Score
1	4	
2	8	
3	10	
4	6	
5	8	
6	8	
7	8	
8	6	
Σ	58	

2. (8 points) Determine whether each improper integral converges or diverges; if it converges, find its value.

(a) $\int_{-2}^6 \frac{1}{(t+2)^{2/3}} dt$

(b) $\int_{15}^{\infty} \frac{2}{x \ln(x)} dx$

3. A company earns revenue at a continuous annual rate of 0.04 (4%) of its net worth. At the same time, the company's payroll obligations amount to 10 million dollars per year, paid out continuously.
- (a) (2 points) Write a differential equation that governs the net worth, W , of the company in millions of dollars.
- (b) (2 points) Determine the initial net worth W_0 that will keep the net worth constant; in other words, determine the value of W_0 for which $W = W_0$ is an equilibrium solution of the differential equation.
- (c) (4 points) Solve the differential equation given that the initial net worth is $W_0 = 100$.
- (d) (2 points) How long will it take for the company to go bankrupt? In other words, how long will it take for the net worth of the company to be zero?

4. (a) (4 points) Find the solution to the following initial value problem:

$$\begin{aligned}x \frac{dy}{dx} + 5y &= 0 \\ y(1) &= 1\end{aligned}$$

(b) (2 points) Verify that your solution satisfies the differential equation and the initial condition.

5. (8 points) Evaluate the following integrals.

(a) $\int \frac{2u}{7-u} du$

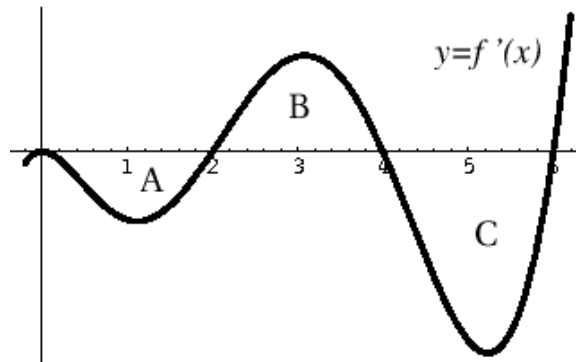
(b) $\int_1^e x^3 \ln(x) dx$

6. On Planet Ψ in Galaxy Ω the acceleration due to gravity is -50 meters per second per second.

(a) (6 points) A rock is thrown at a velocity of 60 meters per second upwards from a height of 8 meters above the surface of Planet Ψ . Find equations for the velocity v and position s of the rock at time t seconds after it is thrown.

(b) (2 points) How high above the surface of the planet does the rock go?

7. Let $f(x)$ be a function satisfying $f(0) = 2$ whose *derivative* is graphed below; the area of region **A** is **3**, the area of region **B** is **4**, and the area of region **C** is **6**.



- (a) (2 points) Find the value of $f(6)$.
- (b) (2 points) Find the average rate of change of f on the interval $[0, 6]$.
- (c) (4 points) Find the absolute (global) maximum value of $f(x)$ on the interval $[0, 6]$, and state where it occurs.

8. (a) (2 points) Sketch the region bounded by the parabola $y = x^2$ and the line $y = 3x$.

(b) (2 points) Set up an integral to compute the area of the region. You do not need to compute the integral.

(c) (2 points) Set up an integral to compute the volume of the solid formed by rotating the region about the x -axis. You do not need to compute the integral.