

Extra Credit (10 points):

The population of trees in a certain forest will, if left alone, experience exponential growth (ie, the rate of increase will be proportional to the population itself). The growth (proportionality) constant in this case is 0.01 when the time  $t$  is given in years.

However, this particular forest is being harvested by a logging company at a constant rate of 100 trees per year. In a show of good faith, the company has agreed to plant saplings in place of some of the harvested trees. This will also happen at a constant rate, say  $S$  trees per year.

Assume further that the initial number of trees in the forest is 1000. [Note: the numbers used in the problem are obviously too small to make sense in reality, but should make your computations easier.]

1. Write a differential equation describing the rate of the change of the population of trees in the forest.

*Solution:*

Notice that we have three factors affecting the tree population here: the growth rate,  $.01P$ , an increasing factor; the logging rate, 100, a decreasing factor; and the replanting rate, constant  $S$ , another increasing factor. Thus the differential equation we want is

$$\frac{dP}{dt} = 0.01P - 100 + S$$

2. By solving this equation, determine the smallest replanting rate  $S$  for which the logging is sustainable (ie, such that the population of trees does not drop to zero). Please box your answer!

*Solution:*

Separating:

$$\frac{dP}{0.01P - 100 + S} = dt$$

Integrating:

$$100 \ln(0.01P - 100 + S) = t + C$$

Solving for P:

$$P = Ae^{0.01t} + 100(100 - S)$$

Using initial condition  $P(0) = 1000$  to get particular solution:

$$1000 = A + 100(100 - S) \Rightarrow A = 100(S - 90)$$

Thus, we get

$$P = 100(S - 90)e^{0.01t} + 100(100 - S)$$

The population of trees is sustainable when the coefficient in front of the exponential,  $100(S - 90)$ , is nonnegative. Thus,  $S \geq 90$  and the smallest sustainable replanting rate is 90 saplings per year.

Note: Beware! Many people got this number through faulty reasoning and thus did not receive credit!