Midterm 1

Math 3C: Precalculus
Instructor: David Lenz
October 24, 2019

Name: Solutions PID: _____

Seat Number: _____

Do not begin this exam until instructed to do so.

When the exam begins, FIRST write your name and PID at the top of each page. **Pages without a name may not be graded.**

There are 8 problems on this exam. You must show the steps you took to arrive at your answer in order to receive full points (unless explicitly stated otherwise). If you need more space for your answer than is provided, write a clear statement that your answer continues on a separate page. Then, using an empty sheet of scratch paper, continue your answer and submit this sheet with your exam. You must tell the TA that you have extra pages when you turn in your exam. If you need to repeat this process for multiple questions, use a separate sheet for each question.

The use of calculators and formula sheets ("cheat sheets") is prohibited on this exam. All phones and similar electronic devices, as well as notes and notebooks must be put away in a closed bag or likewise out of sight.

At the end of the exam, bring your student ID and exam to the front of the hall to be collected.

Problem 1 Let $m(y) = -4y - 5$ and $n(y) = -y + 1$. Where do the graphs of $m(y)$ and $n(y)$ intersect? Write your answer as a coordinate pair.

$m(y) = n(y)$

$-4y - 5 = -y + 1$

$-4y = -y + 6$

$-3y = 6$

$y = -2$

~~n(y) =~~

$n(-2) = -(-2) + 1$

$= 2 + 1$

$= 3$

Intersects at $(-2, 3)$.

Is $m(y)$ an increasing function?

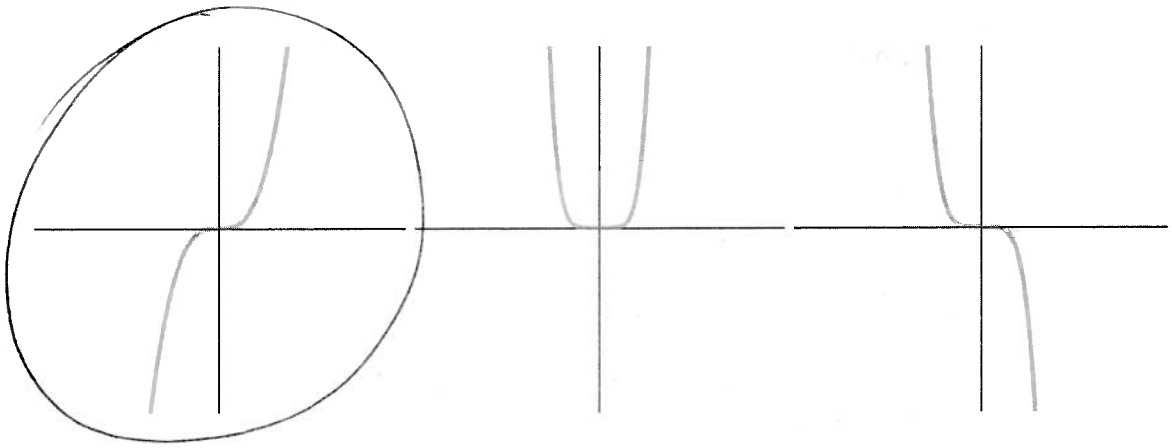
No, the slope is negative

Is $n(y)$ an increasing function?

No, the slope is negative

Problem 2 Suppose $h(t)$ is a function, and the long-run behavior of h is that $h(t)$ tends to ∞ as $t \to \infty$, and that $h(t)$ tends to $-\infty$ as $t \to -\infty$.

Which of the following graphs could possibly represent $h(t)$? *(circle one)*



Which of the following formulas could be the formula for $h(t)$? *(circle one)*

$h(t) = -t^6$ $\quad$ $h(t) = -t^5$ $\quad$ $h(t) = t^3$ $\quad$ $h(t) = t^2$

Problem 3 Let

$$a(z) = \frac{\sqrt[3]{z+2}}{5}.$$

Find a formula for $a^{-1}(z)$.

Let $y = a(z) = \frac{\sqrt[3]{z+2}}{5}$

Solve for z:

$y = \frac{\sqrt[3]{z+2}}{5}$

$5y = \sqrt[3]{z+2}$

$(5y)^3 = z+2$

$(5y)^3 - 2 = z$

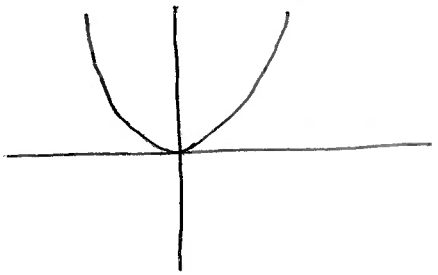
So $a^{-1}(y) = (5y)^3 - 2$

alternatively (changing the variable name)

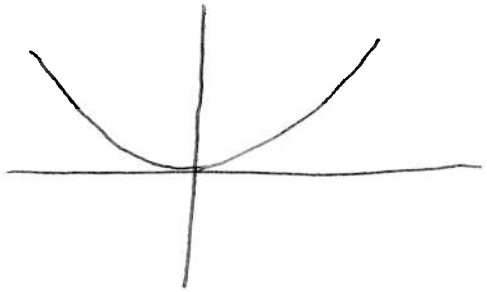
$a^{-1}(z) = (5z)^3 - 2$

Problem 4 Sketch the graphs of the following functions. Each function is a slight modification of the function before it.

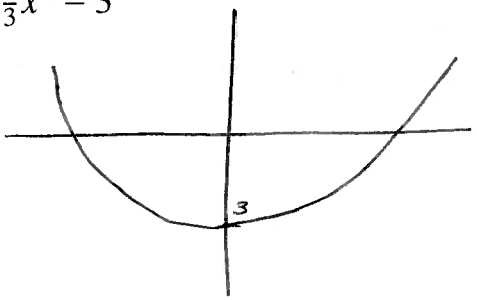
Sketch $f(x) = x^2$



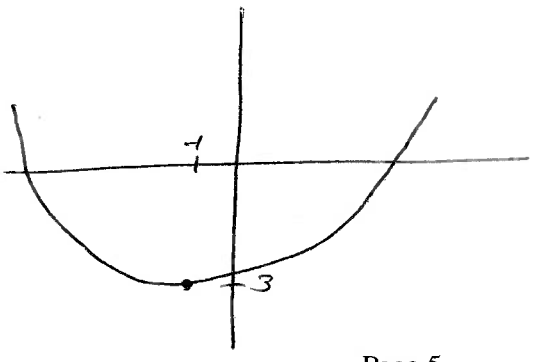
Sketch $f(x) = \frac{1}{3}x^2$



Sketch $f(x) = \frac{1}{3}x^2 - 3$



Sketch $f(x) = \frac{1}{3}(x+1)^2 - 3$



Problem 5 True or False. Write the word "True" or "False" next to each statement. You *do not* need to show your work for this question.

False — If $f(x)$ is some function, then $f(\frac{1}{2}x)$ transforms f with a horizontal stretch (expansion) by a factor of $\frac{1}{2}$.

True — Two lines are parallel if they have the same slope.

False — The function $k(x) = \frac{2}{x^2} - 1$ is one-to-one.

True — The range of a function is the set of all possible outputs of that function.

False — If $f(x)$ is some function, then the graph of $f(x+1)$ is the same as the graph of $f(x)$, but shifted right by one unit.

False — The set $(-2, 1)$ contains the number 1.

False — Two lines are perpendicular if their slopes add up to 0.

False — $f(x) = |x|$ is an odd function.

Problem 6 Let $f(x) = \frac{3x}{x-2}$ and $g(x) = 2x + 2$. Find a formula for $f(g(x))$.

$$f(g(x)) = f(2x+2)$$

$$= \frac{3(2x+2)}{(2x+2)-2} = \frac{6x+6}{2x} = \frac{2(3x+3)}{2x}$$

$$= \frac{3x+3}{x}$$

Problem 7 Let $f(x)$ and $g(x)$ be defined by the following tables. You do not need to show your work for this question.

x	3	2	-1	6	7	0	4
f(x)	2	1	5	-4	0	-1	2

x	8	0	-2	2	6	5	1
g(x)	-3	6	3	4	-1	-5	0

Determine the following:

What is the domain of f? You can write your answer as a list.

3, 2, -1, 6, 7, 0, 4

What is the range of f? You can write your answer as a list.

2, 1, 5, -4, 0, -1

Is $f(x)$ a one-to-one function? Why or why not?

No, $f(3)=2$ and $f(4)=2$, so not every output has a unique input

What is $(g \circ f)(3)$?

$(g \circ f)(3) = g(f(3)) = g(2) = 4$

What is $g^{-1}(0)$?

$g(1)=0 \longrightarrow g^{-1}(0) = 1$.

Problem 8 Solve the equation $|3x - 9| = 3$ for x.

If $|3x-9| = 3$, then

$(3x-9) = 3$ or $-(3x-9) = 3$

$\Rightarrow 3x = 12$ $\quad -3x + 9 = 3$

$\Rightarrow x = 4$ or $x = 2$