Problem 1 (8 points). Consider the function \( f(x) = (0.4)^x \).

(a) What is the domain of \( f(x) \)?

\[ \text{All real numbers} \]

(b) What is the range of \( f(x) \)?

\[ \text{All } y > 0 \]

(c) Determine a formula for the inverse of \( f(x) \).

\[
\begin{align*}
\text{Solve for } y = (0.4)^x & \rightarrow \log_{0.4}(y) = x & \rightarrow f(x) = \log_{0.4}(x) \\
\end{align*}
\]

(d) Does the graph of \( f(x) \) have any asymptotes? Where?

\[ \text{Yes, horizontal asymptote at } y = 0. \]
(e) Does the graph of \( f(x) \) have any vertical intercepts? If so, write your answer as a coordinate pair.

\[ \text{Yes, at } (0, 1) \]

(f) Does the graph of \( f(x) \) have any horizontal intercepts? If so, write your answer(s) as a coordinate pair.

\[ \text{No} \]

(g) Sketch the graph of \( f(x) \). Label any intercepts or asymptotes.

\[ \\
\text{H Asymptote} \\
y = 0 \\
(0, 1) \\
\]
Problem 2 (8 points). Consider the function \( g(x) = -2(0.4)^{x+3} \).

(a) What is the domain of \( g(x) \)?

\[ \text{Shift left by 3, shifts domain of \( f(x) \) left by 3.} \]

(b) What is the range of \( g(x) \)?

\[ g(x) \] is \( f(x) \) stretched vertically by power of 2 and reflected vertically.

(c) Determine a formula for the inverse of \( g(x) \).

\[ y = \frac{-x}{2} = 0.4^{x+3} \]
\[ \log_{0.4} \left( \frac{-x}{2} \right) = x + 3 \]
\[ x = \log_{0.4} \left( \frac{-y}{2} \right)^3 \]
\[ g^{-1}(x) = \log_{0.4} \left( \frac{x}{2} \right)^{-3} \]

(d) Does the graph of \( g(x) \) have any asymptotes? Where?

Yes, horizontal asymptote at \( y = 0 \).

(e) Does the graph of \( g(x) \) have any vertical intercepts? If so, write your answer as a coordinate pair.

Yes \[ g(0) = -2(0.4)^{0+3} = -2(0.4)^3 \]
Intercept = \((0, -2(0.4)^3)\)

(f) Does the graph of \( g(x) \) have any horizontal intercepts? If so, write your answer(s) as a coordinate pair.

No
(g) Sketch the graph of \( g(x) \). Label any intercepts or asymptotes.
Problem 3 (8 points). Consider the function $h(x) = \log_6(x)$.

(a) What is the domain of $h(x)$?

$$\mathbb{R} \ (0, \infty)$$

(b) What is the range of $h(x)$?

All real numbers

(c) Determine a formula for the inverse of $h(x)$.

Solve for $y = \log_6(x) \rightarrow x = 6^y \rightarrow h^{-1}(x) = 6^x$

(d) Does the graph of $h(x)$ have any asymptotes? Where?

Yes, vertical asymptote at $x = 0$

(e) Does the graph of $h(x)$ have any vertical intercepts? If so, write your answer as a coordinate pair.

No

(f) Does the graph of $h(x)$ have any horizontal intercepts? If so, write your answer(s) as a coordinate pair.

$(1, 0)$
(g) Sketch the graph of \( h(x) \). Label any intercepts or asymptotes.
Problem 4 (8 points). Consider the function \( k(x) = 2 \log_6(x - 2) + 4 \).

(a) What is the domain of \( k(x) \)?

\( k(x) \) is like \( h(x) \), but shifted right by 2 and vertically stretched/shifted.

Domain of \( h(x) \) was \((0, \infty)\), so domain of \( k(x) \) is \([2, \infty)\).

(b) What is the range of \( k(x) \)?

\( k(x) \) is like \( h(x) \), but vertically shifted up by 4 and vertically stretched by 2, and horizontally shifted.

Since range of \( h(x) \) was \((-\infty, 0)\), range of \( k(x) \) is also \((-\infty, 0]\).

(c) Determine a formula for the inverse of \( k(x) \).

Solve for \( x \):

\[
y = 2 \log_6(x - 2) + 4
\]

\[
\Rightarrow \frac{x - 4}{2} = \log_6(x - 2)
\]

\[
\sqrt{x} = 6^{\frac{x - 4}{2}} + 2
\]

\[
k^{-1}(x) = 6^{\frac{x - 4}{2}} + 2
\]

(d) Does the graph of \( k(x) \) have any asymptotes? Where?

Yes, vertical asymptote at \( x = 2 \). This is the asymptote of \( h(x) \) shifted right by 2.

(e) Does the graph of \( k(x) \) have any vertical intercepts? If so, write your answer as a coordinate pair.

\( \text{No} \)

(f) Does the graph of \( k(x) \) have any horizontal intercepts? If so, write your answer(s) as a coordinate pair.

\( \text{Not Graded} \)

\( \text{sorry!} \)
(g) Sketch the graph of $k(x)$. Label asymptotes, but you do not need to label intercepts for this problem.

- Transformations
  - Shift right 2
  - Stretch vertically by 2
  - Shift up 4
Problem 5 (8 points). Consider the function \( s(x) = \cos(x) \).

(a) What is the domain of \( s(x) \)?

\[ \text{All real numbers} \]

(b) What is the range of \( s(x) \)?

\[ [-1, 1] \]

(c) Determine a formula for the inverse of \( s(x) \) (this is not a trick question, see Chapter 6.3).

\[ \cos^{-1}(x) \quad \text{or} \quad s^{-1}(x) \text{ does not exist} \]

(d) Does the graph of \( s(x) \) have any asymptotes? Where?

\[ \text{No} \]

(e) Does the graph of \( s(x) \) have any vertical intercepts? If so, write your answer as a coordinate pair.

\[ \text{Yes.} \quad s(0) = \cos(0) = 1 \quad \rightarrow \quad (0, 1) \]

(f) Does the graph of \( s(x) \) have any horizontal intercepts? If so, write your answer(s) as a coordinate pair.

\[ \text{Yes. Find all points such that } \cos(x) = 0. \]

\[ \rightarrow x = \frac{\pi}{2} + 2\pi \cdot n \]

\[ \rightarrow x = \frac{3\pi}{2} + 2\pi \cdot n \quad \text{where } n \text{ is an integer} \]
(g) Sketch the graph of \( s(x) \). Label intercepts or asymptotes in a clear way.
Problem 6 (8 points). Consider the function $r(x) = -\cos(3x) + 5$.

(a) What is the domain of $r(x)$?

All real numbers

(b) What is the range of $r(x)$?

$[4, 6] \xrightarrow{\text{range of } s(x) \text{ shifted up by 5}}$ [Will not be assessed on final]

(c) Determine a formula for the inverse of $r(x)$.

No inverse exists or On the interval $[0, \frac{2\pi}{3}]$, $r^{-1}(x) = \cos^{-1}(5-x) \cdot \frac{1}{3}$

(d) Does the graph of $r(x)$ have any asymptotes? Where?

No

(e) Does the graph of $r(x)$ have any vertical intercepts? If so, write your answer as a coordinate pair.

Yes. $r(0) = -\cos(3\cdot0) + 5 = -\cos(0) + 5 = -1 + 5 = 4 \Rightarrow (0, 4)$

(f) Does the graph of $r(x)$ have any horizontal intercepts? If so, write your answer(s) as a coordinate pair.

No, range is $[4, 6]$, so $r(x)$ never touches horizontal axis.
(g) Sketch the graph of $r(x)$. Label intercepts or asymptotes in a clear way.

Period is $\frac{2\pi}{3}$ because $r(x)$ has a horizontal stretch by $\frac{1}{3}$.