Problem 1 (3 points). Let $\theta = 240^\circ$.

(a) What quadrant is $\theta$ in?

![3rd Quadrant]

(b) What is the reference angle of $\theta$?

![60°]

(c) What are $\sin(\theta)$ and $\cos(\theta)$?

$$\sin(240^\circ) = -\sin(60^\circ) = -\frac{\sqrt{3}}{2}$$
$$\cos(240^\circ) = -\cos(60^\circ) = -\frac{1}{2}$$

Problem 2 (3 points). Let $\theta = -210^\circ$.

(a) What quadrant is $\theta$ in?

![2nd Quadrant]

(b) What is the reference angle of $\theta$?

$30^\circ$
(c) What are \( \sin(\theta) \) and \( \cos(\theta) \)?

\[
\begin{align*}
\sin(-210^\circ) &= \sin(30^\circ) = \frac{1}{2} \\
\cos(-210^\circ) &= -\cos(30^\circ) = -\frac{\sqrt{3}}{2}
\end{align*}
\]

**Problem 3** (3 points). Let \( \theta = -\frac{3\pi}{4} \).

(a) What quadrant is \( \theta \) in?

(b) What is the reference angle of \( \theta \) (in radians)?

\[
\frac{\pi}{4}
\]

(c) What are \( \sin(\theta) \) and \( \cos(\theta) \)?

\[
\begin{align*}
\sin\left(-\frac{3\pi}{4}\right) &= -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \\
\cos\left(-\frac{3\pi}{4}\right) &= -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}
\end{align*}
\]

**Problem 4** (3 points). Let \( \theta = \frac{15\pi}{4} \).

(a) What quadrant is \( \theta \) in?

(b) What is the reference angle of \( \theta \) (in radians)?

\[
\frac{\pi}{4}
\]

(c) What are \( \sin(\theta) \) and \( \cos(\theta) \)?

\[
\begin{align*}
\sin\left(\frac{15\pi}{4}\right) &= -\sin\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2} \\
\cos\left(\frac{15\pi}{4}\right) &= \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}
\end{align*}
\]
Problem 5 (4 points). True or False.

**False**

Let $\theta$ be an angle such that $\sin(\theta) > 0$. It is possible that $\theta$ is in the third quadrant.

**True**

Let $\gamma$ be an angle such that $\sin(\gamma) < 0$. It is possible that $\gamma$ is in the fourth quadrant.

**False**

Let $\beta$ be an angle such that $\cos(\beta) < 0$. It is possible that $\beta$ is in the fourth quadrant.

**True**

Let $\alpha$ be an angle such that $\cos(\alpha) < 0$ and $\sin(\alpha) > 0$. It is possible that $\alpha$ is in the second quadrant.

Problem 6 (4 points). Let $\alpha$ be an angle such that $\sin(\alpha) = \frac{3}{5}$, and $\alpha$ is in the second quadrant. What is $\cos(\alpha)$? *(HINT: recall that $\cos(\theta)^2 + \sin(\theta)^2 = 1$ for any angle $\theta$)*

Pythagorean Identity:

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

$$\Rightarrow \cos^2(\theta) + \left(\frac{3}{5}\right)^2 = 1$$

$$\Rightarrow \cos^2(\theta) + \frac{9}{25} = \frac{25}{25}$$

$$\Rightarrow \cos^2(\theta) = \frac{16}{25}$$

$$\Rightarrow \cos(\theta) = \pm \sqrt{\frac{16}{25}}$$

$$\Rightarrow \cos(\theta) = \pm \frac{4}{5}$$

**BUT** $\theta$ is in the second quadrant, so $\cos(\theta) < 0$. Thus $|\cos(\theta)| = -\frac{4}{5}$.
Problem 7 (6 points). Fill in the coordinate of each point on the unit circle below. Note: You will be required to do this with no notes on quizzes/exams in the future!
**Problem 8** (2 points). Find the coordinates of the point on a circle of radius 6 at an angle of $\frac{\pi}{3}$ radians

\[
\begin{align*}
X &= r \cdot \cos(\theta) = 6 \cdot \cos\left(\frac{\pi}{3}\right) = 6 \cdot \frac{1}{2} = 3 \\
Y &= r \cdot \sin(\theta) = 6 \cdot \sin\left(\frac{\pi}{3}\right) = 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}
\end{align*}
\]

\[(x, y) = (3, 3\sqrt{3})\]

**Problem 9** (4 points). The point $(-7, -7\sqrt{3})$ lies on the circle of radius 14. At what angle around the circles does this point lie?

\[
\begin{align*}
X &= -7 = r \cdot \cos(\theta) = 14 \cdot \cos(\theta) \\
\Rightarrow \quad -7 &= 14 \cos(\theta) \\
\Rightarrow \quad \cos(\theta) &= -\frac{1}{2}
\end{align*}
\]

\[
\begin{align*}
Y &= -7\sqrt{3} = r \cdot \sin(\theta) = 14 \sin(\theta) \\
\Rightarrow \quad -7\sqrt{3} &= 14 \sin(\theta) \\
\Rightarrow \quad \sin(\theta) &= -\frac{\sqrt{3}}{2}
\end{align*}
\]

What angle satisfies both $\cos(\theta) = -\frac{1}{2}$ and $\sin(\theta) = -\frac{\sqrt{3}}{2}$? 

$\theta = 240^\circ$ or $300^\circ$ 

$\theta = 120^\circ$ or $240^\circ$
Problem 10 (8 points). Compute the following:

(a) \( \tan(135^\circ) \)

\[
\tan(135^\circ) = \frac{\sin(135^\circ)}{\cos(135^\circ)} = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1
\]

(b) \( \csc\left(\frac{\pi}{6}\right) \)

\[
\csc\left(\frac{\pi}{6}\right) = \frac{1}{\sin\left(\frac{\pi}{6}\right)} = \frac{1}{\frac{1}{2}} = 2
\]

(c) \( \cot\left(\frac{4\pi}{3}\right) \)

\[
\cot\left(\frac{4\pi}{3}\right) = \frac{\cos\left(\frac{4\pi}{3}\right)}{\sin\left(\frac{4\pi}{3}\right)} = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\frac{1}{2}} \cdot \frac{2}{-\sqrt{3}} = -\frac{1}{\sqrt{3}}
\]

(d) \( \sec(180^\circ) \)

\[
\sec(180^\circ) = \frac{1}{\cos(180^\circ)} = \frac{1}{-1} = -1
\]
Problem 11 (8 points). Simplify the following expressions into a single trig function with no fractions:

(a) \( \cot(\theta) \sin(\theta) \)

\[
\cot(\theta) \sin(\theta) = \frac{\cos(\theta)}{\sin(\theta)} \cdot \sin(\theta) = \cos(\theta)
\]

(b) \( \sec(\theta) \csc(\theta) \)

Not graded

In Week 10, we learn that \( \sin(2\theta) = 2\sin(\theta)\cos(\theta) \).

Using this:

\[
\sec(\theta) \csc(\theta) = \frac{1}{\sin(\theta)} \cdot \frac{1}{\cos(\theta)} = \frac{1}{\sin(\theta)\cos(\theta)}
\]

\[
= \frac{1}{\frac{1}{2} \sin(2\theta)} = 2 \cdot \frac{1}{\sin(2\theta)} = 2 \csc(\theta)
\]

(c) \( \csc(\theta) \cos(\theta) \)

\[
csc(\theta) \cos(\theta) = \frac{1}{\sin(\theta)} \cdot \cos(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \cot(\theta)
\]

(d) \( \frac{1-\sin^2(\theta)}{\sin(\theta)^2} \)

Since \( \sin^2(\theta) + \cos^2(\theta) = 1 \), it's true that \( \cos^2(\theta) = 1 - \sin^2(\theta) \).

So

\[
\frac{1-\sin^2(\theta)}{\sin^2(\theta)} = \frac{\cos^2(\theta)}{\sin^2(\theta)} = \cot^2(\theta)
\]

[OPTIONAL]

Survey Questions.

1. Do you find the lectures to go:
   
   too fast    too slow    roughly the right speed

2. How do you feel about Midterm 2?

   Not confident    Somewhat not confident    No Idea    Somewhat confident    Confident