

What would happen to sea level if Greenland's ice sheet melted? (part 1)

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MATH 111B

Mathematical Modeling

La Jolla, CA, 4 February 2019

Recent news

February 4, 2019. A third of Himalayan ice cap doomed, finds report

<https://www.theguardian.com/environment/2019/feb/04/a-third-of-himalayan-ice-cap-doomed-finds-shocking-report>

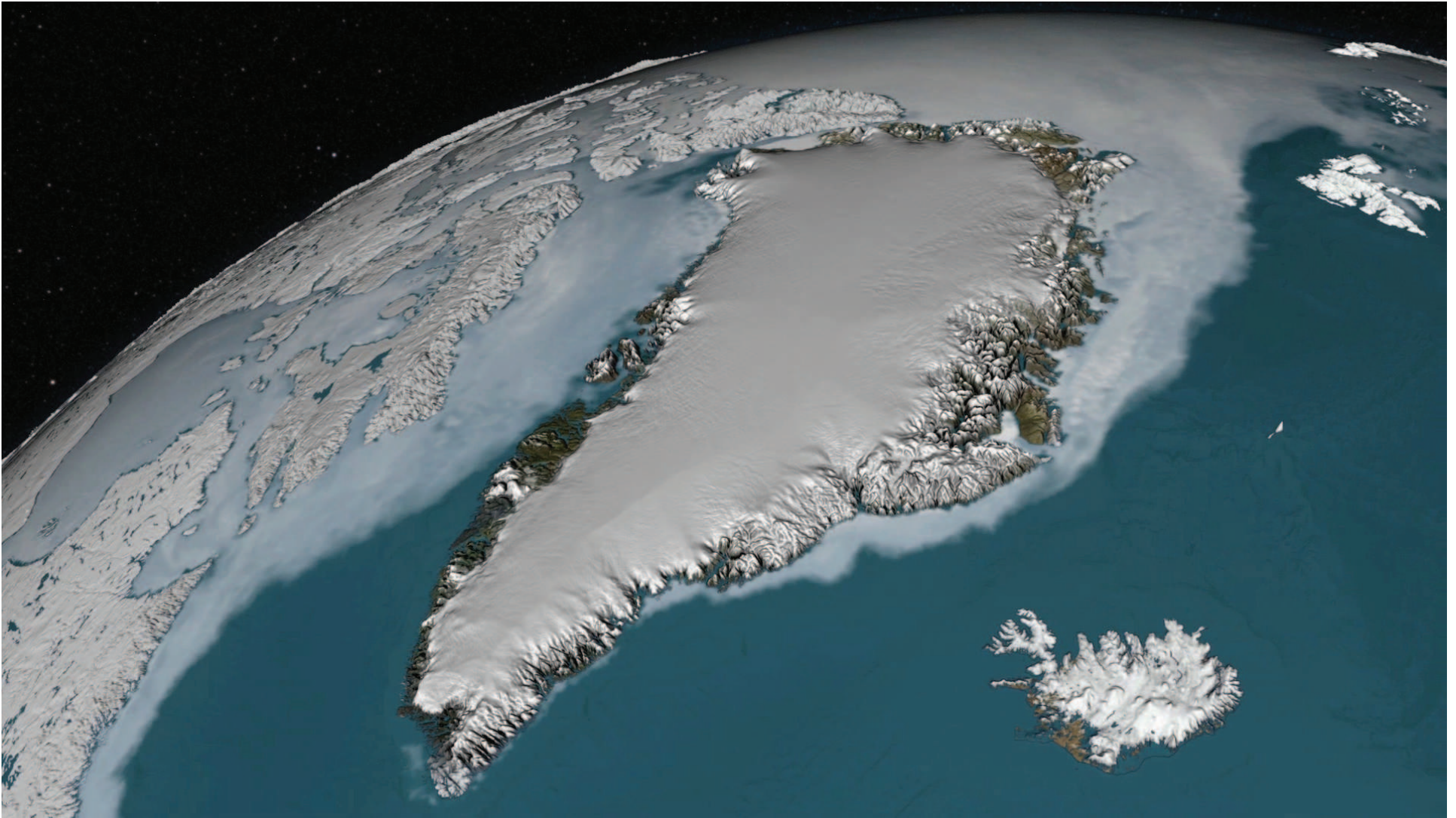
January 18, 2019. Antarctica's ice is melting faster, raising risk of sea level rise

<https://www.cbsnews.com/news/antarctica-ice-melting-six-times-greater-study-finds/>

January 22, 2019. Greenland's Ice Sheet Is Melting Fast

<https://www.bloomberg.com/news/videos/2019-01-22/greenland-s-ice-sheet-is-melting-fast-video>

NASA, *Greenland's Ice Layers Mapped in 3D*



<http://www.nasa.gov/content/goddard/nasa-data-peers-into-greenlands-ice-sheet>

The Greenland ice sheet

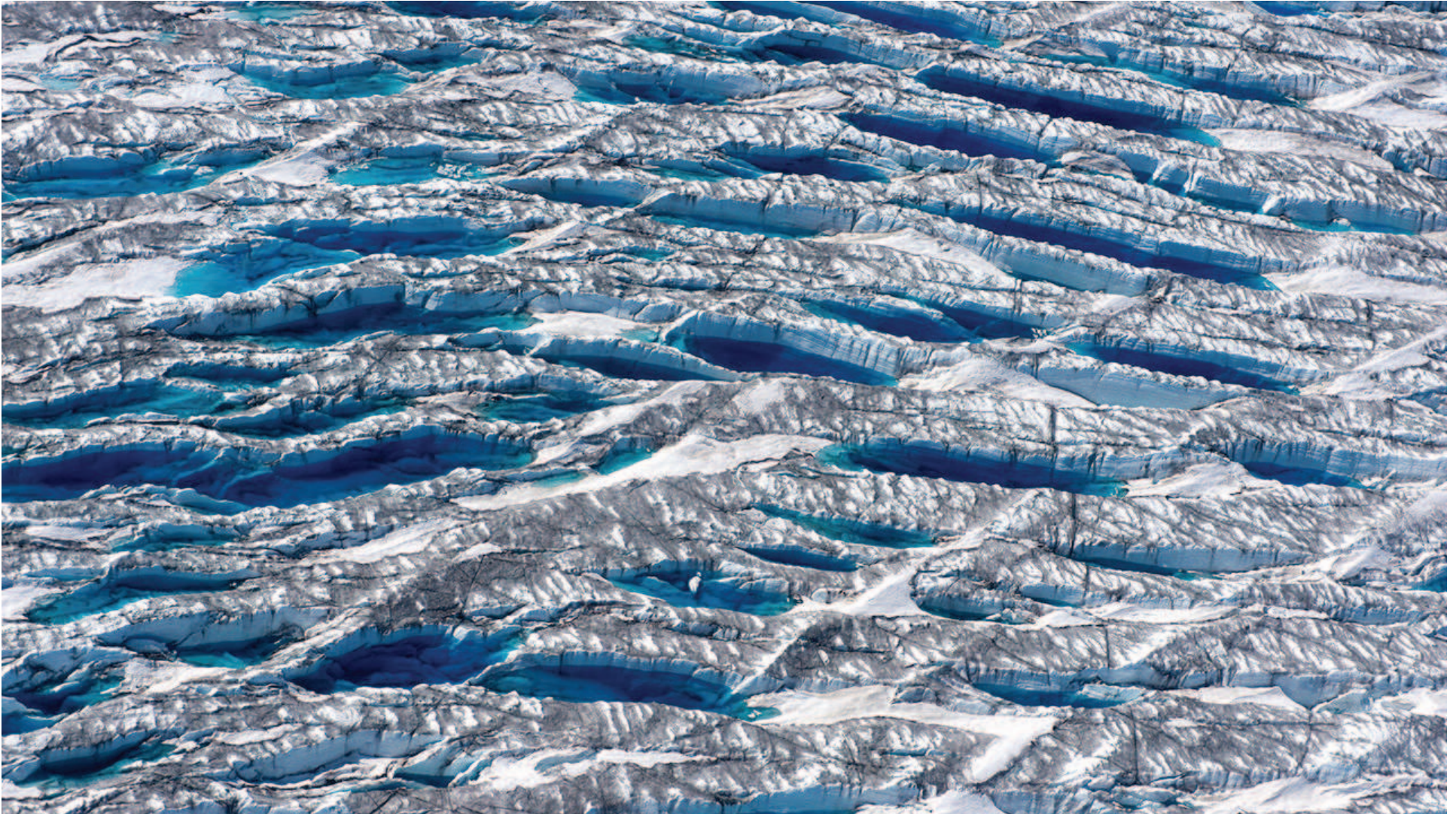
... is Earth's second largest after Antarctica's ice sheet(s).

... has volume approximately $2.9 \times 10^6 \text{ km}^3$.

<http://polarportal.dk/en/groenlands-indlandsis/nbsp/viden-om-groenlands-indlandsis/>

... and has been melting.

Meltwater lakes on the Greenland ice sheet



Note the algae and dirt darkening the ice, lowering its albedo.

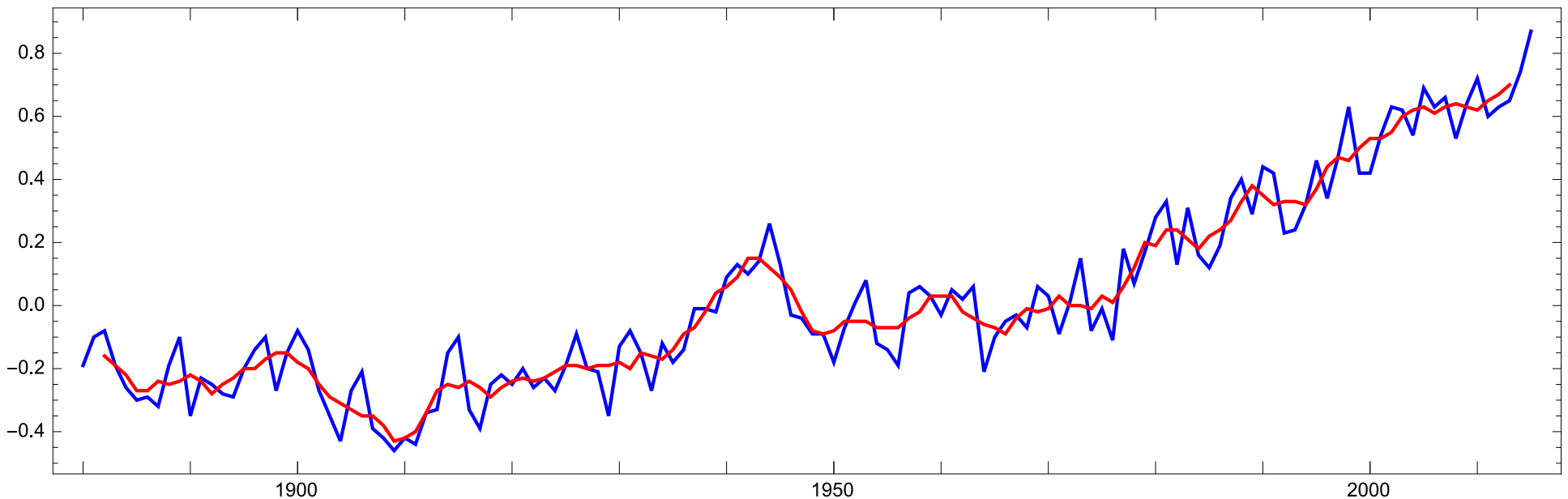
A. LeWinter, Extreme Ice Survey, <http://extremeicesurvey.org/>.

Global temperatures have been rising

Climate Lab Book temperature spiral:

http://www.climate-lab-book.ac.uk/files/2016/09/spiral_aug2016.gif

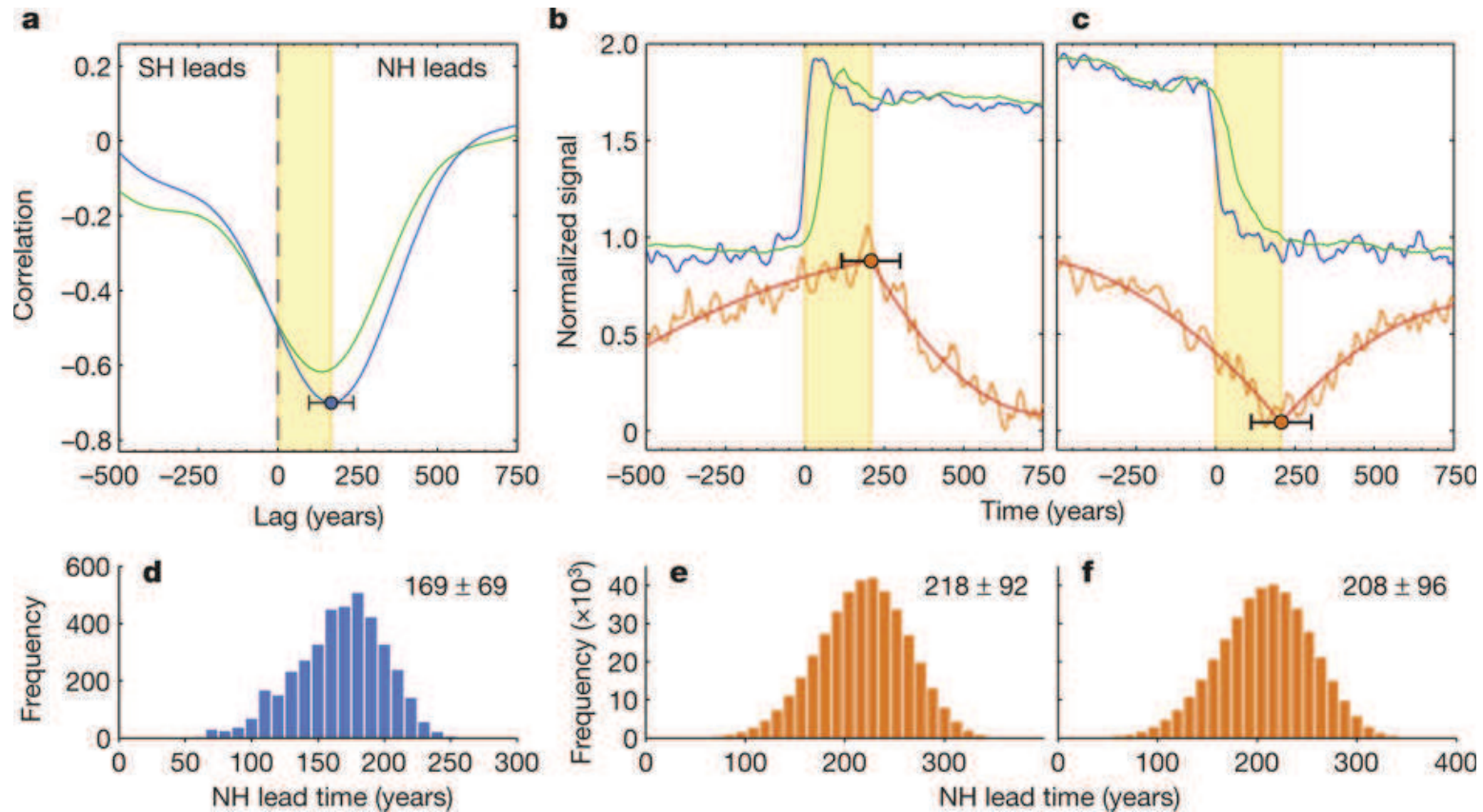
Global temperature difference from 1951–1980 average, in °C; **annual** and **5 year moving** average:



<http://climate.nasa.gov/vital-signs/global-temperature/>

The northern hemisphere historically reacts before the southern

There is an approximately 200 year lag between temperature changes in Greenland and Antarctica over the last 68kyr:



WAIS Divide Project, "Precise inter-polar phasing of abrupt climate change during the last ice age", Nature 520 (2015) 661–665.

Venice, December 2008



R. Melania, "A flooding of Venice" (26 April 2012).

<https://ralucamelaniaphotography.wordpress.com/2012/04/26/a-flooding-of-venice/>

High tide flooding in Fort Lauderdale, FL



S. Wdowinski, "Flooding events increase on beaches", UM News Special Report (2016).
<http://climate.miami.edu/the-complex-climate/flooding-events-increase-on-beaches/>

King tide flooding in La Jolla, CA, January 12, 2017



King tide flooding in La Jolla, CA, January 12, 2017



King tide flooding in La Jolla, CA, January 12, 2017



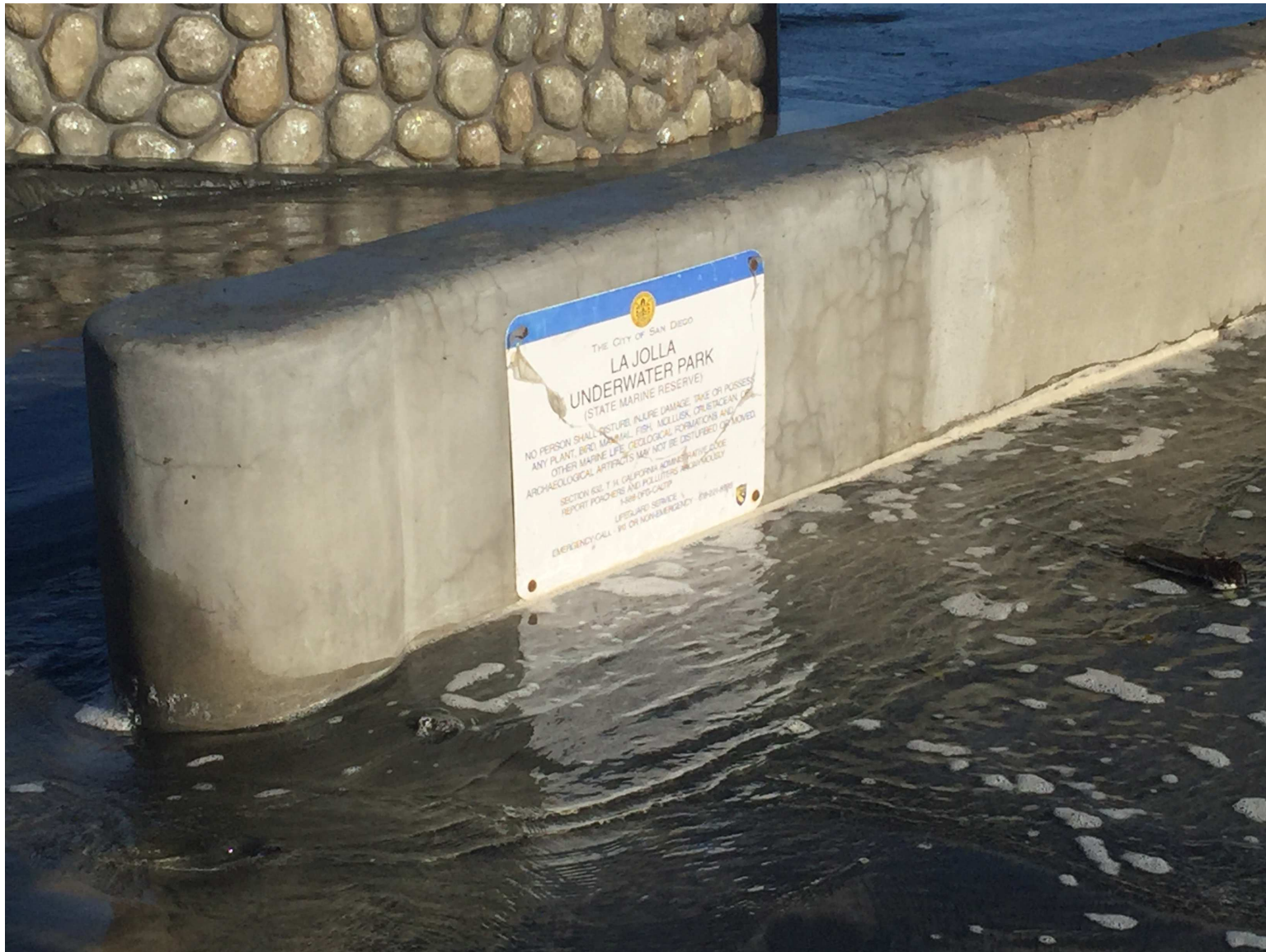
King tide flooding in La Jolla, CA, January 12, 2017



King tide flooding in La Jolla, CA, January 21, 2019



King tide flooding in La Jolla, CA, January 21, 2019

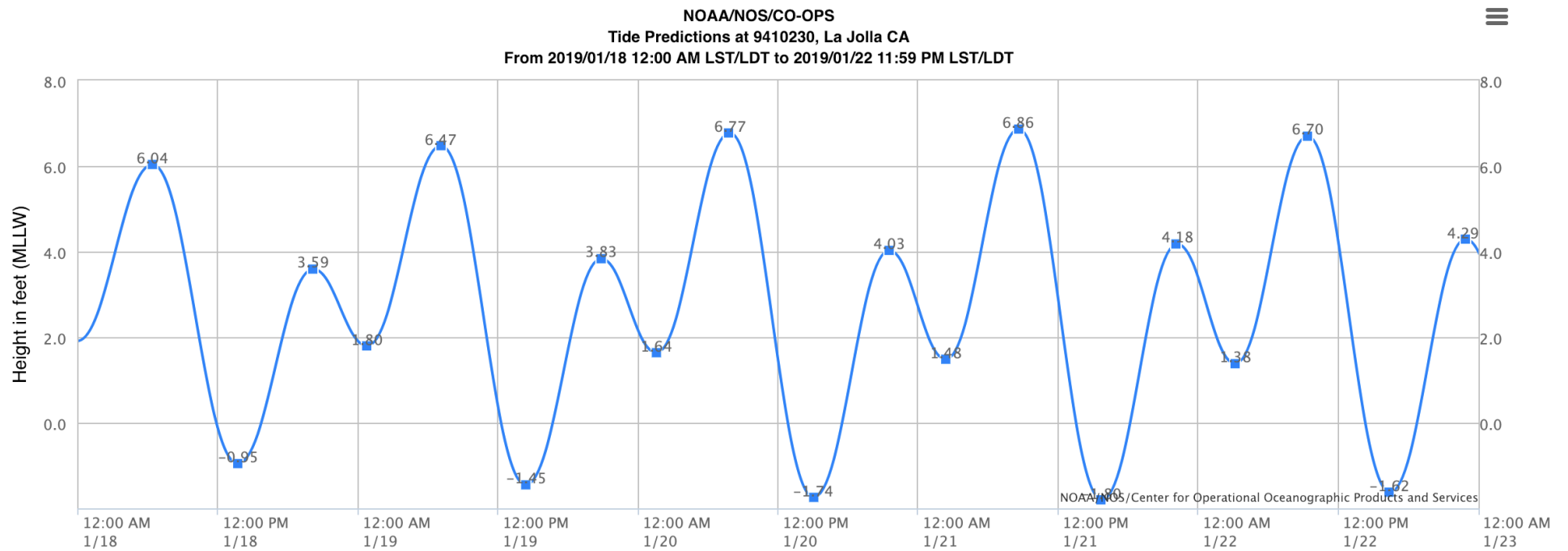


King tide flooding in La Jolla, CA

January 12, 2017. Avenida de la Playa [[video 1](#)] [[video 2](#)]

January 21, 2019. Avenida de la Playa [[video](#)]

King tide height in La Jolla, CA, January 21, 2019



Arithmetic

Density of ice is approximately $0.9 \times$ density of water.

So $2.9 \times 10^6 \text{ km}^3$ of ice melts into approximately $V = 2.61 \times 10^6 \text{ km}^3$ of water.

About 71% of the Earth's surface is covered by water, approximately $A = 3.6 \times 10^8 \text{ km}^2$.

So the water from the melting of the Greenland ice sheet would raise the global sea level by

$$h \approx \frac{2.61 \times 10^6 \text{ km}^3}{3.6 \times 10^8 \text{ km}^2} = 0.00725 \text{ km} \approx 7 \text{ m}.$$

Greenland (Kalaallit Nunaat)

Greenland is an autonomous country within the Kingdom of Denmark.

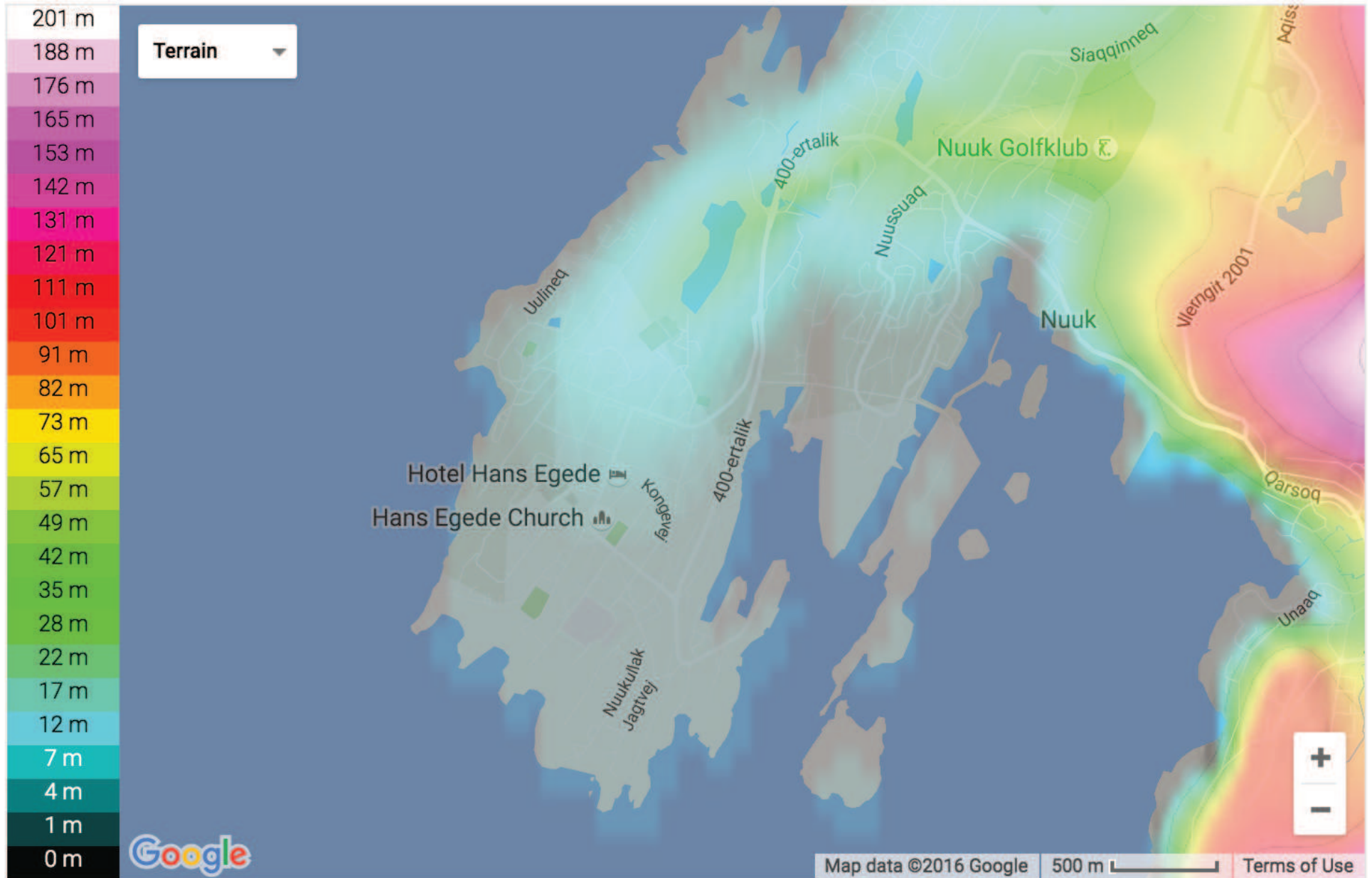
Its population is approximately 56K, mostly Inuit.

About 1/3 of the population lives in the capital city, Nuuk, on the southwest coast of Greenland.

Nuuk, Greenland

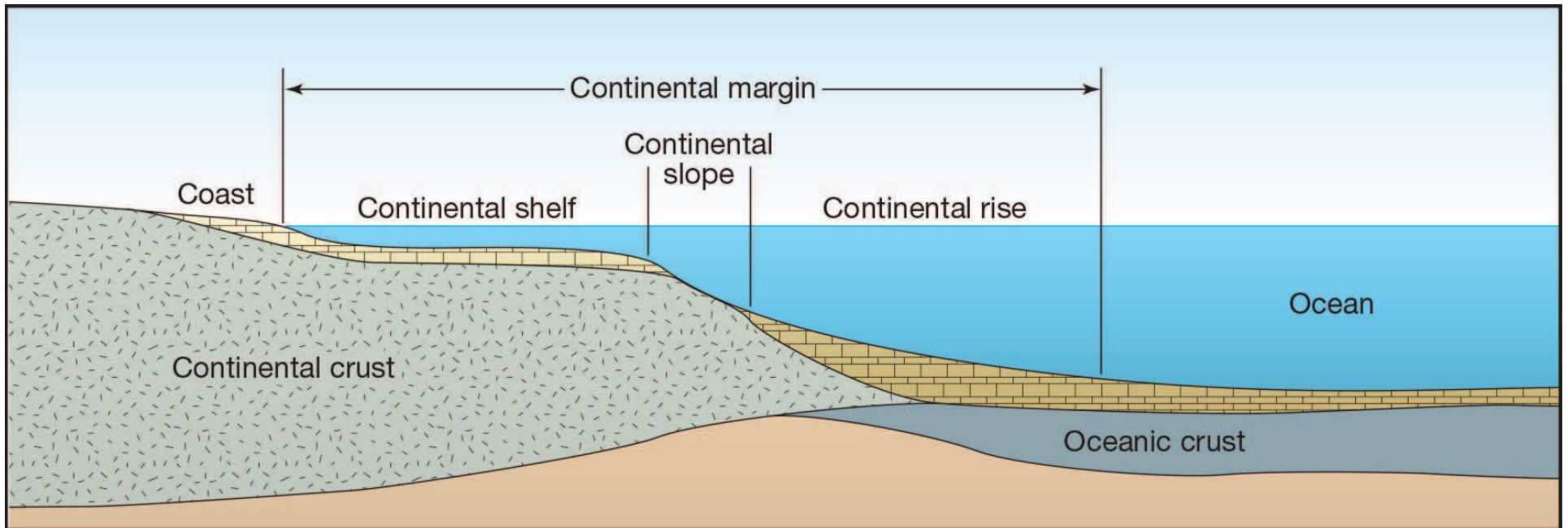


Nuuk, Greenland



The coast isn't vertical

... so as sea level rises the ocean covers more surface area, reducing the height of the volume of melt water.

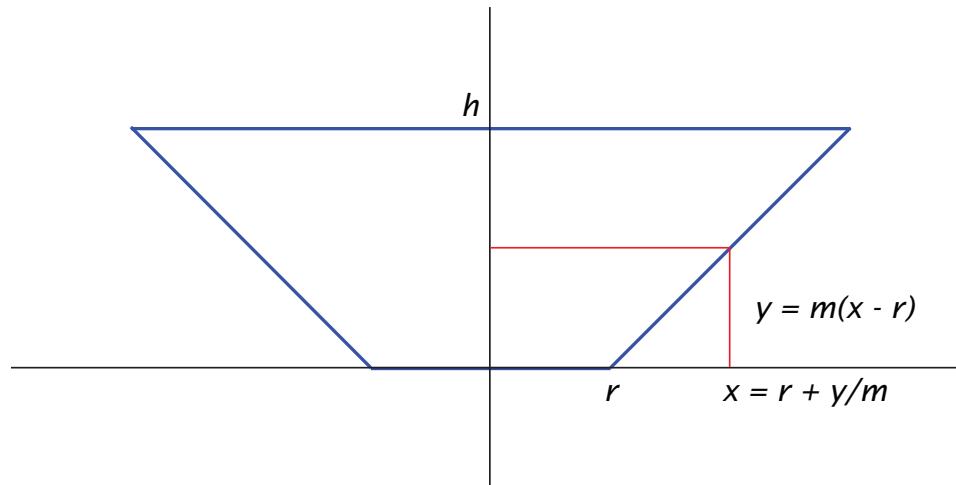


The slope of the continental shelf is approximately $m = 0.002$.

R. Nagel, Encyclopedia of Landforms and other geologic features (Thomson Gale 2004).

Geometry

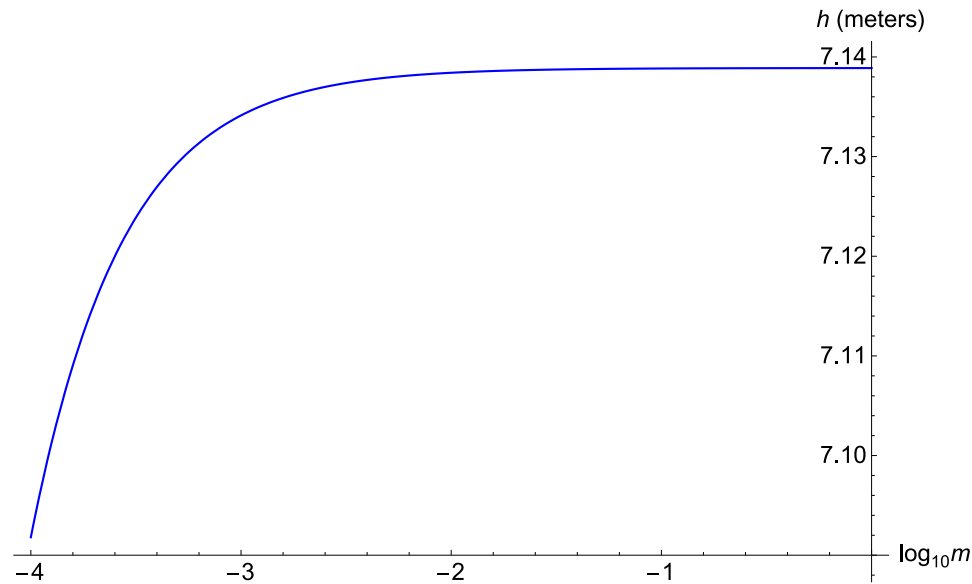
Approximate the ocean surface as a circle of radius r given by $\pi r^2 = A$.



$$V = \int_0^h \pi x^2 dy = \pi \int_0^h \left(r + \frac{y}{m}\right)^2 dy = \pi \left(r^2 h + 2r \frac{h^2}{2m} + \frac{h^3}{3m^2}\right)$$

This lets us solve for h as a function of m ; notice that **Descartes'** Rule of Signs tells us there is only one real root.

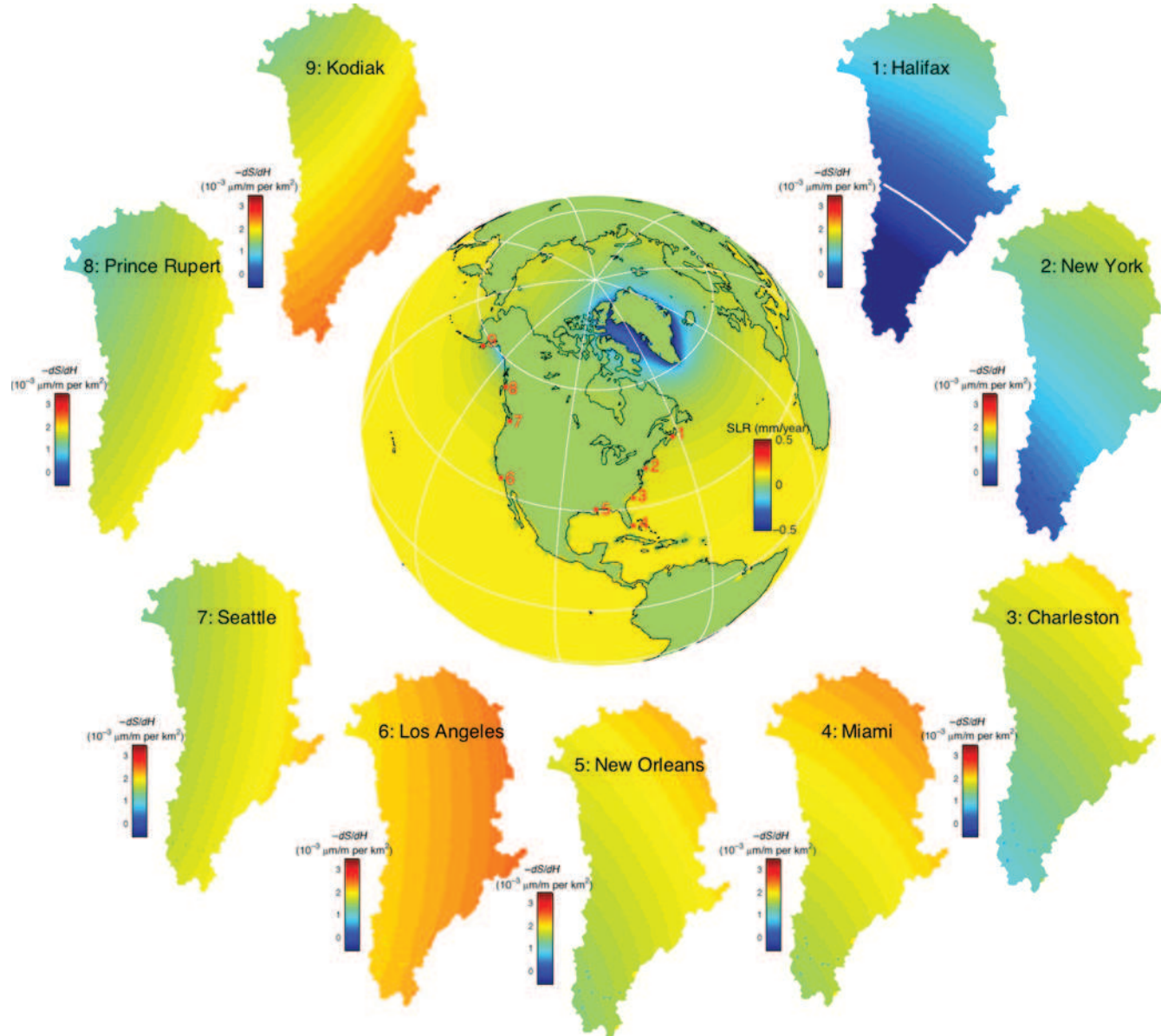
Coastal slope has an insignificant effect



$$V = \pi r^2 \cdot h + \pi r \cdot \frac{h}{m} \cdot h + \frac{\pi}{3} \left(\frac{h}{m}\right)^2 \cdot h$$

For $r \gg h/m$, i.e., $m \gg h/r \approx 7 \times 10^{-7}$, the first term on the right hand side dominates; this is the vertical coast approximation. **The ocean is really wide!**

Sea level rise is not uniform



E. Larour, E. R. Irwins and S. Adhikara, "Should coastal planners have concern over where land ice is melting?", *Science Advances* 3 (2017) e1700537.