MATH 168A 22 April 2019

Random Walk Algorithms: Homework 3

1. Show that the heat kernel we derived in lecture,

$$u(t,x) = \frac{1}{\sqrt{4\pi\alpha t}} e^{-x^2/(4\alpha t)},$$

satisfies the heat equation:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}.$$

2. For $x \in \mathbb{Z}/16\mathbb{Z}$, let

$$u_{0,x} = \frac{1}{56} \begin{cases} x & \text{if } x \in \{0,1,2,3,4,5,6,7\}; \\ x-8 & \text{otherwise.} \end{cases}$$

Define a random walk on $\mathbb{Z}/16\mathbb{Z}$ by $\vec{u}_{t+1} = P\vec{u}_t$, where

$$P_{ij} = \frac{1}{4} \begin{cases} 2 & \text{if } i = j; \\ 1 & \text{if } i - j \equiv \pm 1 \pmod{16}; \\ 0 & \text{otherwise.} \end{cases}$$

Write code to compute and plot $u_{t,x}$ for each $t \in \{0,1,2,4,8,16\}$, connecting the points $(x, u_{t,x})$ and $(x+1, u_{t,x+1})$ for $x \in \{0,1,\ldots,14\}$ to make a piecewise linear function on $[0,15] \subset \mathbb{R}$. Plot these six functions in the same diagram.