MATH 168A 22 May 2019

Random Walk Algorithms: Homework 7

As we saw in lecture, a 2-CNF expression may or may not have a satisfying assignment. The goal of this week's homework is to understand how the probability of a *random* 2-CNF expression having a satisfying assignment depends upon the number of variables and the number of clauses.

1. Show that every 2-CNF expression in $n \in \mathbb{N}$ variables with only a single clause has a satisfying assignment.

2. What is the maximum number of distinct clauses, K_n, a 2-CNF in $n \in \mathbb{N}$ variables can have? Show that a 2-CNF with K_n distinct clauses has no satisfying assignment.

3. Write a piece of code to generate a 2-CNF with k distinct clauses in $n \in \mathbb{N}$ variables, uniformly at random from all such 2-CNF expressions. Your code need only handle $1 \leq k \leq K_n$.

4. Using Papadimitriou's algorithm for finding a satisfying assignment of an $n \in \mathbb{N}$ variable 2-CNF expression that we discussed in class (or some other algorithm for finding one) write a piece of code to estimate (because the algorithm might fail to find a satisfying assignment even when one exists) the probability that a random 2-CNF expression in $n \in \mathbb{N}$ variables with $1 \leq k \leq K_n$ distinct clauses has a satisfying assignment. Plot your estimate as a function of k for $n = 10$, $n = 20$ and $n = 40$.