168A PRACTICE MIDTERM

In problem 1, write pseudocode for your algorithm; you can also describe in words what it is doing. In problems 2 and 3, showing your work in addition to the final answer might earn some partial credit in case the latter is not exactly right.

1. Construct a randomized algorithm to approximate the integral of an arbitrary function $f : [0, 1] \to [0, 1]$. Assume that the function is provided as a subroutine which returns $f(x)$ when passed argument x; also assume you have access to a random number generator random() that returns a uniformly random number in the interval $[0, 1]$, *e.g.*, $x \leftarrow$ random() means that $x \in [0, 1]$, sampled from the constant probability density function. Design your algorithm so that it has probability at least 2/3 of giving an estimate for the integral that is correct to one decimal place.

2. Consider a random walk $\{X_t \in \mathbb{Z} \mid t \in \mathbb{N}\}$ with transition probabilities

$$P_{xy} = \begin{cases} 1/2 & \text{if } |x - y| = 1; \\ 0 & \text{otherwise,} \end{cases}$$

and initial state $X_0 = 0$.

a. Suppose $X_2 = 0$. What is the probability that $X_t \neq 0$ for any $0 < t < 2$, *i.e.*, that given the random walk has returned to 0 at $t = 2$, that this is the first return?

b. Suppose $X_4 = 0$. What is the probability that $X_t \neq 0$ for any $0 < t < 4$, *i.e.*, that given the random walk has returned to 0 at $t = 4$, that this is the first return?

c. Suppose $X_6 = 0$. What is the probability that $X_t \neq 0$ for any $0 < t < 6$, *i.e.*, that given the random walk has returned to 0 at $t = 6$, that this is the first return?

3. a. Let G be a graph with 4 vertices and adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

What is the transition probability matrix for the random walk on G that for each vertex v has equal transition probability to each vertex connected to v, and transition probability 0 to every other vertex?

b. Find a probability distribution on the vertices of G that is unchanged by the random walk evolution defined in problem 3.a.