Math 10C - Fall 2009 - Final Exam

Name: _____

Student ID: _____

Section time: _____

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books or telephones. You may use a "cheat sheet" of notes which should be a page, front only.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 10 questions which are worth 150 points. You have 180 minutes to complete the test.

Question	Score	Maximum
1		15
2		20
3		13
4		17
5		15
6		15
7		15
8		15
9		10
10		15
Total		150

Problem 1. [*15 points*]

At what point (x, y, z) on the plane $x + 2y - z = 5$ does the minimum of the function

$$f(x, y, z) = x^2 + 2y^2 + (z + 1)^2$$

occur?

Problem 2. [*20 points.*]

Consider the function

$$f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy.$$

(i) [8] Find the critical points of the function.

(ii) [8] Determine the nature of the critical points (local min/local max/saddle).

(iii) [4] Does the function $f(x, y)$ have a global minimum or a global maximum?

Problem 3. [*13 points*] Consider the function

$$f(x, y) = \ln(xy^2) - \frac{2x}{y}$$

(i) [8] Compute the second order Taylor polynomial of f around $(1, 1)$.

(ii) [5] Find the tangent plane to the graph of f at the point $(1, 1, -2)$.

Problem 4. [*17 points.*]

Find the global minimum and global maximum of the function

$$f(x, y) = x^2 + y^2 - 2x - 2y + 4$$

over the closed disk

$$x^2 + y^2 \leq 8.$$

Problem 5. [*15 points*]
Consider the function

$$f(x, y) = 1 + x^2 + y^2.$$

(i) [4] Draw the contour diagram of f labeling at least three levels of your choice.

(ii) [4] Compute the gradient of f at $(1, -1)$ and draw it on the contour diagram of part (i).

(iii) [3] Does the function f have a global minimum? If no, why not? If yes, what is the minimum value?

(iv) [4] Draw the graph of the function f.

Problem 6. [*15 points*]
Consider the function

$$f(x,y) = e^{-3x+2y}\sqrt{2x+1}\,.$$

(i) [5] Calculate the gradient of f at $(0,0)$.

(ii) [5] Find the directional derivative of f at $(0,0)$ in the direction $\mathbf{u} = \frac{i+j}{\sqrt{2}}$.

(iii) [5] What is the unit direction for which the rate of increase of f at $(0, 0)$ is maximal?

Problem 7. [*15 points*]
Consider the planes

$$x + 2y - z = 1,\ x + 4y - 2z = 3.$$

(i) [4] Find normal vectors to the two planes.

(ii) [6] Are the two planes parallel? Are they perpendicular?

(iii) [5] Find a vector parallel to the line of intersection of the two planes.

Problem 8. [*15 points.*]

Consider the function

$$w = u^2 v\, e^{-v}$$

and assume that

$$u = x^2 - 2xy,\ \ v = -x + 2\ln y.$$

Calculate the values of the derivatives

$$\frac{\partial w}{\partial x} \text{ and } \frac{\partial w}{\partial y}$$

at the point $(x, y) = (1, 1)$.

Problem 9. [*10 points*]

You deposit $\$1,000$ into your savings account *every year* for the next 10 years. You make the first deposit on January 1, 2010, and the last deposit on January 1, 2019. The interest rate for the account is $r = 10\%$ per year. How much money will there be in your account at the end of the 10^{th} year, on December 31, 2019?

Express your answer in the simplest closed form. You don't need to evaluate the powers that may appear in the final expression.

Problem 10. [*15 points*]

The outcome x of a certain experiment has values between 0 and $\frac{\pi}{2}$, with probability distribution function

$$p(x) = \sin x, \text{ for } 0 \leq x \leq \frac{\pi}{2}.$$

(i) [4] Calculate the cumulative distribution function.

(ii) [4] Calculate the median outcome of the experiment.

(iii) [7] Possibly using integration by parts, calculate the mean outcome of the experiment.