Math 103A Fall 2005 Homework 1

Due 10/3/04 in class

(All exercise and page numbers refer to Gallian, 6th edition).

1. Do Gallian Section 0, #8,14,16, 22, 38, 48, 50, 52.

Hints:

for #38, use the modulo 7 check digit as in Example 6, page 9 in the text. A "transposition error" is the interchange of two adjacent digits. The case where the check digit is transposed with the next-to-last digit requires a separate analysis.

for # 52: don't try proof by induction. Instead examine these integers modulo 4.

2. Find (by inspection) at least 3 different choices of integers s and t satisfying the equation $1 = 7s + 11t$. Look for a pattern. Make a conjecture about what the set of all possible pairs s, t of integers satisfying the equation is. It should be clear from your description that there are infinitely many such pairs (but you don't need to prove your conjecture.)

3. The Fibonacci numbers are the sequence of numbers $f_1, f_2, f_3, \ldots$ defined by setting $f_1 = 1$, $f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for all $n \geq 3$. This sequence begins $1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$.

Perhaps it will surprise you to learn that there is a formula for the nth fibonacci number. Prove that

$$f_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}$$

for all $n \geq 1$.