Math 103A Fall 2005 HW 3/Exam 1 review sheet

HW Due 10/17/05 in class

All exercise and page numbers refer to Gallian, 6th edition.

0. These exercises are suggestions for extra practice at home (or in section) and are *not to be turned in!* The ones in bold should be looked over before Exam 1.

Gallian Section 3, #**17**, **21**, **29**, **33**, **51**

Gallian Section 4, # **1**, **3**, **7**, 13, 27, 37, **47**, 59

1. Do Gallian Section 3, #8, 10, 22, 24

2. Do Gallian Section 4, #10, 22, 32, 40, 52, 56, 60

Review Sheet for Exam 1

The Exam will cover Chapters 0-3 and the beginning of Chapter 4; more specifically, everything covered in class up to and including the lecture on Monday 10/10/05.

0.1 definitions to know

GCD, LCM, relatively prime, equivalence relation, equivalence class, injective/surjective/bijective function, group, Abelian group, Cayley table of a group, subgroup, order of a group, order of an element, cyclic subgroup/cyclic group, generator of a cyclic group, center of a group, centralizer of an element.

0.2 The most important groups to know

1. The integers $\mathbb{Z}$ under addition (and similarly, $\mathbb{Q}, \mathbb{R}, \mathbb{C}$ under addition)
2. The nonzero rational numbers $\mathbb{Q} \setminus \{0\}$ under multiplication (and similarly, $\mathbb{R} \setminus \{0\}, \mathbb{C} \setminus \{0\}$ under multiplication)
3. $\mathbb{Z}_n$ (the integers under addition modulo n), for any $n \geq 1$.
4. $U(n)$ (the integers which are relatively prime to n, under multiplication modulo n), for any $n \geq 2$.
5. D_n, the dihedral group of order $2n$, which is the symmetry group of a regular n-gon in the plane.
6. $\mathrm{GL}(2, F)$ and $\mathrm{SL}(2, F)$, where F is any of the following: $\mathbb{Q}, \mathbb{R}, \mathbb{C}$, or $\mathbb{F}_p$ for a prime p.
7. Groups defined by a Cayley table (as in Exercise 3.17)

For all of these groups, you should be able to (i) know generally how to show it satisfies the group axioms; (ii) multiply (or add, if the operation is called addition) any two elements of the group; (iii) find the inverse of any element in the group; (iv) find the centralizer of an element of the group; (v) decide if the group is Abelian; (vi) find the order of any element of the group; (vii) find the cyclic subgroup generated by any element of the group.

0.3 Theorems you should know how to prove

1. The identity of a group is unique
2. Left and right cancellation hold in a group
3. the inverse of an element in a group is unique
4. $\langle a \rangle$, $Z(G)$, $C(a)$ are subgroups of G, for any group G and $a \in G$.

0.4 Theorems you should know what they say and how to use

1. Division Algorithm
2. GCD is a linear combination
3. Using the Euclidean algorithm to find $\gcd(a, b)$ and integers x, y with
$$ax + by = \gcd(a, b).$$
4. Fundamental Theorem of Arithmetic
5. Two-step subgroup test (Theorem 3.2 in the book)
6. Criterion for $a^i = a^j$ in a cyclic group $\langle a \rangle$ (Theorem 4.1 in the book)

0.5 Homework Review

You should review the homework exercises in Problem Sets 1 and 2, and look over the bold practice problems on Problem Set 3.