Math 103A Fall 2005 HW 8 (last one!)

HW Due *Friday* 12/2/05 in class

All exercise and page numbers refer to Gallian, 6th edition.

0. These exercises are suggestions for extra practice at home (or in section) and are *not to be turned in!*

Gallian Chapter 10, #9, 15, 21, 25, 35

Gallian Chapter 11, #5, 9, 13, 17, 19

1. Gallian Chapter 10, #4, 18, 20, 30, 32

2. Gallian Chapter 11, #2, 12, 20, 28, 32

Hint for #20: The question should read "Suppose that G is a finite Abelian group that has exactly one subgroup H with $|H| = d$ for each divisor d of $|G|$. Show that G is cyclic." To approach this problem, I suggest the following proof of the contrapositive. Think about Corollary 1 and 2 in Chapter 8 on page 156. Use these to show that if G is *not* cyclic, then $G \cong \mathbb{Z}_{a_1} \oplus \mathbb{Z}_{a_2} \oplus \cdots \oplus \mathbb{Z}_{a_n}$ where $\gcd(a_i, a_j) = d > 1$ for some pair a_i, a_j. Then show that the group $\mathbb{Z}_{a_i} \oplus \mathbb{Z}_{a_j}$ has more then one element of order d. Use this to find more than one element of order d in G.

Hint for #32: Calculate the order of G and the order of each of the elements in G. The question "What is the isomorphism class of G" means find positive numbers a_i such that $G \cong \mathbb{Z}_{a_1} \oplus \mathbb{Z}_{a_2} \oplus \cdots \oplus \mathbb{Z}_{a_n}$.