Math 103A Fall 2005 Exam 2 review sheet

You should go over your notes, and the homework exercises. If there were any homework exercises for Chapters 4-8 you didn't do or didn't do completely correctly, go over them and make sure you understand them now. You can also go over the odd, "extra practice" exercises. The following list of topics is meant to give you an idea of what I consider most important among the things we have done since Exam 1, but it is not necessarily a complete list of all topics that could appear. Chapter 8 is fair game, up through the material in the lecture on 11/2/05. The homework involving Chapter 8 will not be returned before the exam, so use the suggested extra odd problems in Chapter 8 on Homework #6 to check your understanding.

1 Chapter 4, second half

•Understanding Theorems 4.2 and 4.3.
•Finding the order of an element in a cyclic group (Theorem 4.2).
•Finding all subgroups of a cyclic group (Theorem 4.3).
•Finding all generators of a cyclic group (Corollary 2).
•Definition of the Euler-phi function $\phi(n)$.

2 Chapter 5

•Definition of the symmetric group S_n.
•Changing a permutation in array notation to cycle notation and vice versa.
•understanding how to multiply and find inverses for permutations in either notation.
•Finding the order of a permutation in disjoint cycle form (Theorem 5.1).
•Understanding what Theorem 5.5 says, and definition of even and odd permutations.
•Writing any permutation α as a product of 2-cycles (transpositions), and using this to decide if α is even or odd.

•Definition of alternating group A_n. Knowing the orders of S_n and A_n.
•Finding a group of permutations which is isomorphic to the dihedral group D_n (without proof).
•understanding the proof that the center of S_n is trivial for $n \geq 3$ (exercise 5.46).

3 Chapter 6

•Definition of isomorphism from one group to another.
•Basic properties of isomorphisms (Theorem 6.2 and Theorem 6.3)— you don't necessarily have to memorize all of these— just remember the principle that an isomorphism between groups means they are the"same", just that the elements have been renamed, so all properties of the two groups must correspond.
•Know what Cayley's theorem says (Theorem 6.1) and the basic idea behind the proof (but you won't have to reproduce the proof.)
•Deciding if two groups are isomorphic or not, and proving it. To prove that two groups $G, \overline{G}$ are isomorphic, you must construct an explicit function $\phi : G \to \overline{G}$ and show it satisfies the properties of the definition. To prove that two groups are not isomorphic, possible techniques include showing one is abelian and the other isn't; showing one is cyclic and the other isn't; or showing for some d that the two groups have a different number of elements of order d.
•Proof that all cyclic groups of order n are isomorphic to $\mathbb{Z}_n$ (Example 2).

4 Chapter 7

•Definition of left and right cosets of a subgroup H of a group G. Finding the left cosets for explicit examples.
•Basic properties of left cosets (Lemma p. 138), especially that (1) all left cosets have the same size, and (2) every element of G belongs to exactly one of the distinct left cosets (in other words the distinct left cosets partition G.)
•Understanding what Lagrange's theorem says (Theorem 7.1), and the basic idea of the proof (it follows from the two basic facts about left cosets mentioned above.)
•Understanding what the corollaries of Lagrange's theorem say and being able to use in problems.

•Using Fermat's little theorem to give a test for primality (see class notes and the exercise on HW #6).
•Understanding the classification of groups of order p (Corollary 3) and how it follows from Lagrange's Theorem.
•Understanding the classfication of groups of order $2p$, where p is an odd prime (Theorem 7.2, you don't need to know the proof.)

5 Chapter 8

•Definition of external direct product of groups. Being able to multiply and find inverses in a direct product.
•Finding the order of an element in a direct product (Theorem 8.1).
•Calculating how many elements of order d a direct product has. See example in class notes 10/31 or Example 4 in the text.
•Calculating how many cyclic subgroups of order d a direct product has. See example in class notes 10/31 or Example 5 in the text.
•Understanding when a direct product is cyclic (Theorem 8.2 and Corollary 1).
•Decomposing Z_n and $U(n)$ into smaller direct products (Corollary 2 of Theorem 8.2, Theorem 8.3 and its Corollary).
•Formula for the Euler phi-function $\phi(n)$ (Class notes 11/2).