Math 103A Fall 2006 HW 4/Exam 1 review sheet

HW Due 10/20/05 in class

All exercise and page numbers refer to Gallian, 6th edition. The ones in bold should be looked over before Exam 1.

0. These exercises are suggestions for extra practice at home (or in section) and are *not to be turned in!* Gallian Chapter 7, #**3**, **5**, **7**, **9**, **15**, 19, 23.

1. Do Gallian Chapter 7, #**6**, **8**, **10**, 18, 22, 24, 26.

Review Sheet for Exam 1

The Exam will cover Chapters 0-3, the beginning of Chapter 4 (up through the top of p. 78), and the beginning of Chapter 7 (up through the bottom of page 142) EXCEPT topics not covered in class (such as check digit schemes introduced in chapter 0). In terms of lectures, this is everything covered in class up to and including the lecture on Friday 10/13/06.

0.1 definitions to know

GCD, LCM, relatively prime, equivalence relation, equivalence class, function, 1-1 function, onto function, group, Abelian group, Cayley table of a group, subgroup, order $|G|$ of a group G, order of an element in a group, the center $Z(G)$ of a group G, the centralizer $C(g)$ of an element $g \in G$, the cyclic subgroup $\langle x \rangle$ generated by an element $x \in G$, cyclic group, generator of a cyclic group, left and right cosets of a subgroup H of a group G.

0.2 The most important groups to know

1. The integers $\mathbb{Z}$ under addition (and similarly, $\mathbb{Q}, \mathbb{R}$under addition)
2. The nonzero rational numbers $\mathbb{Q} \setminus \{0\}$ under multiplication (and similarly, the nonzero real numbers $\mathbb{R} \setminus \{0\}$ under multiplication)
3. $\mathbb{Z}_n$ (the group of classes $[n]$ under addition modulo n), for any $n \geq 2$.
4. $U(n)$ (the group of classes $[k]$ such that $\gcd(k, n) = 1$, under multiplication modulo n), for any $n \geq 2$.
5. D_n, the dihedral group of order $2n$, which is the symmetry group of a regular n-gon in the plane.
6. $\mathrm{GL}(2, \mathbb{R})$ the group of 2×2 matrices with nonzero determinant under matrix multiplication, and similarly the group $\mathrm{GL}(2, \mathbb{Q})$

For all of these groups, you should have a general understanding of them such that you are able to (i) have some idea why it satisfies the group axioms; (ii) multiply (or add, if the operation is called addition) any two elements of the group; (iii) find the inverse of any element in the group; (iv) find the centralizer of an element of the group; (v) decide if the group is Abelian; (vi) find the order of the group or the order of an element of the group; (vii) find the cyclic subgroup generated by any element of the group; (viii) decide if the group is cyclic.

0.3 Theorems you should know how to prove

1. $Z(G)$ and $C(a)$ are subgroups of G, for any group G and $a \in G$. The book proves this for $Z(G)$ and I proved it in class for $C(a)$. (I also proved $Z(G)$ is a subgroup in class in a different way.)

2. Lagrange's Theorem (assuming as given the basic properties of cosets.)

3. Fermat's Little Theorem (assuming Lagrange's Theorem)

0.4 Theorems you should know what they say and how to use

1. Two-step subgroup test (Theorem 3.2 in the book); if you prefer the one-step subgroup test, fine.

2. Criterion for $a^i = a^j$ in a cyclic group $\langle a \rangle$ (Theorem 4.1 in the book)

3. Properties of Cosets (Lemma p. 138 in the book). You don't have to remember all of the proofs, but know the main idea of how I did it in class (using that left cosets of H in G are the same as equivalence classes of the equivalence relation on G defined by $x \sim y$ if $x^{-1}y \in H$.)

0.5 Homework Review

You should review the homework exercises in Problem Sets 1, 2, and 3, and look over the bold practice problems on Problem Set 4. Sometimes homework problems reappear on exams.