

Math 103A Fall 2006 HW6 with Exam 2 review

HW Due **11/17/05** in class

All exercise and page numbers refer to Gallian, 6th edition. Look over the **bold** exercises before Exam II, although this homework is not due until long after.

0. These exercises are suggestions for extra practice at home (or in section) and are *not to be turned in!*

Gallian Chapter 5, #**5**, **7**, **9**, **11**, **15**, 37, 41

Gallian Chapter 8, #5, 9, 11, 17

1. Do Gallian Chapter 5, #**6**, 8, **12**, **18**, 26, **28**, 36, 38, 46

2. Do Gallian Chapter 8, #2, 4, 12, 16.

1 Review Sheet for Exam II

Here is a list of some of the major topics we have covered since the first exam, to help you study the most important points. You are still responsible for any topics we covered before Exam I. Also, you are responsible for any topic we covered in class, whether or not it is listed here. You are also responsible for knowing the homework exercises.

2 Chapter 4, second half

- Have a general understanding of Theorem 4.2 and be able to find the order of an element in a cyclic group, especially Z_n . (You shouldn't have to memorize the formula for the order of an element— just be able to find the order in examples.)
- Understand what Theorem 4.3 says, and be able to find all of the subgroups of a finite cyclic group.
- Understand how to find all generators of a cyclic group (Corollary 2 and 3 p. 77).
- Definition of the Euler-phi function $\phi(n)$.

3 Chapter 5

- Definition of the symmetric group S_n .
- Changing a permutation in array notation to cycle notation and vice versa.
- understanding how to multiply and find inverses for permutations in either notation.
- Finding the order of a permutation in disjoint cycle form (Theorem 5.1).
- Understanding what Theorem 5.5 says, and definition of even and odd permutations.
- Writing any permutation α as a product of 2-cycles (transpositions), and using this to decide if α is even or odd.
- Definition of alternating group A_n . Knowing the orders of S_n and A_n .

4 Chapter 9

- Definition of Normal Subgroup; understanding how to check a subgroup is normal using normal subgroup test.
- Every subgroup of an Abelian group is normal; understand why.
- Examples of Normal subgroups: The rotation subgroup is normal in D_n . In fact if H is any subgroup $H \subset G$ where the index $|G : H| = 2$, then H is normal in G . $SL(2, \mathbb{R})$ is normal in $GL(2, \mathbb{R})$. A_n is normal in S_n (again, it has index 2.)
- Definition of a Factor group G/H . Look over the proof of Theorem 9.2 (though you won't be expected to reproduce it): in particular, why H must be normal in G in order for G/H to make sense. Be able to multiply and find inverses of elements of a factor group. Look over the various examples of factor groups I gave in class.

- The most important example of a factor group: for $n \geq 1$, $\mathbb{Z}/n\mathbb{Z}$ is none other than \mathbb{Z}_n . Understand why \mathbb{Z}_n can be thought of in this way as a factor group.
- Be able to prove simple facts about factor groups as were addressed in your exercises. Two examples are: A factor group of a cyclic group is cyclic, and a factor group of an Abelian group is Abelian.
- Understand how to find the order of an element of a factor group, as addressed in your exercises. (For example, 9.14.)

5 Chapter 10

- Know the definition of a homomorphism $\phi : G \rightarrow G'$, the kernel $\ker \phi$, and the image $\phi(G)$.
- Know some examples of homomorphisms; I gave quite a few in class.
- Know some of the basic properties of homomorphisms as given in Theorem 10.1, Theorem 10.2, and the Corollary. But only the ones I actually covered in class. Review the proofs of these basic facts.
- One of the more important basic facts is the following: If $\phi : G \rightarrow G'$ is a homomorphism, then $\phi(a) = \phi(b)$ if and only if a and b are in the same left coset of $\ker \phi$, i.e. if and only if $a(\ker \phi) = b(\ker \phi)$. Thus ϕ “collapses” the elements of G in packets of size $\ker \phi$. This is the picture you should have in your head of a homomorphism.
- Know the statement of the first isomorphism theorem and given some homomorphism $\phi : G \rightarrow G'$, be able to write down the isomorphism that the first isomorphism theorem provides.
- Know that any infinite cyclic group is isomorphic to \mathbb{Z} and any cyclic group of order n is isomorphic to \mathbb{Z}_n . Review the proofs of these results I gave in class, since this is a good way to review the first isomorphism theorem.
- Understand how to find and work with homomorphisms $\phi : \mathbb{Z}_m \rightarrow \mathbb{Z}_n$, as I discussed in class on Monday 10/30, and as was addressed in several of your exercises in Chapter 10. The most important fact is that such a homomorphism ϕ is completely determined once you know that $\phi([1]) = [d]$, since this then forces $\phi([a]) = [ad]$ for all integers a . Moreover, you should be able to decide which choices of d give you homomorphisms (the issue is to check that the formula $\phi([a]) = [ad]$ really defines a function.)