Math 103B Winter 2006 HW 8

HW Due Friday 3/17/06 in class

All exercise and page numbers refer to Gallian, 6th edition.

0. These exercises are suggestions for extra practice at home or in section and are not to be turned in.

Gallian Chapter 20, #17, 19

Gallian Chapter 31, #3, 5, 7, 13

1. Gallian Chapter 20, #6.

2. Let $g(x) = x^3 + 2x + 1 \in \mathbb{Q}[x]$. Let $\alpha \in \mathbb{C}$ be a root of g over the complex numbers (do not try to find α explicitly; you don't need to in order to do this problem.) You may quote the theorem we proved in class on Wednesday 3/8.

(a). Find the minimal polynomial of α over $\mathbb{Q}$.

(b). Explain how you know that

$$K = \mathbb{Q} + \mathbb{Q}\alpha + \mathbb{Q}\alpha^2 = \{a + b\alpha + c\alpha^2 | a, b, c \in \mathbb{Q}\}$$

is a subfield of $\mathbb{C}$.

(c). Find explicit $a, b, c \in \mathbb{Q}$ such that $\alpha^{-1} = a + b\alpha + c\alpha^2$ (note that some such expression must exist since K is a field.)

(d). Find the degree of the field extension $\mathbb{Q} \subset \mathbb{Q}(\alpha)$. Recall that this is defined to be the dimension of $\mathbb{Q}(\alpha)$ as a vector space over $\mathbb{Q}$.

3. Let $G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$.

Use this matrix to form an error correcting code V. The code words in V are all vectors in $\mathbb{Z}_2^7$ which are in the image of the linear transformation ϕ which takes a row vector $v \in \mathbb{Z}_2^4$ to the row vector $vG \in \mathbb{Z}_2^7$. While writing down all explicitly all 16 code words may make some parts of this problem easier, it is possible to answer all the parts without resorting to this tedious task.

(a). Find the weight of the code V.

(b). Let t be the maximum number of errors the code can correct. Find t. Find an explicit example of $t+1$ errors made to a code word $v \in V$, where these errors are not correctable.

(c). Let s be the maximum number of errors the code can detect. Find s. Find an explicit example of $s+1$ errors made to a code word $v \in V$, where these errors are not detectable.

(d). Find the parity check matrix H for this code. Suppose you receive the word 1101110 as part of a message, and you know that at most one error was made in the transmission. Using H, decide if there is an error in the received word, and if so, decide what the sent word actually was.