MATH 109 WINTER 2007 HOMEWORK 3, DUE 1/26/07 IN CLASS

(All exercise and page numbers refer to Eccles.)

1. Reading and practice

Finish reading Chapter 5 of Eccles, and do the end of chapter 5 exercises for practice if you haven't already. Read Chapter 6 and do the end of chapter 6 exercises for practice. Do not hand any of these in.

2. Exercises to submit on Friday 1/26

The first two problems are about the Fibonacci numbers. Recall that the nth Fibonacci number u_n is defined inductively by putting $u_1 = 1$, $u_2 = 1$, and $u_{n+1} = u_n + u_{n-1}$ for $n \geq 2$. In these problems, do not use the Binet formula.

1. Prove by induction that u_n is even if 3 divides n and u_n is odd if 3 does not divide n.

2. Prove by induction that for all $n \geq 1$, $(u_{n+1})^2 - u_{n+2}\, u_n = (-1)^n$.

#19 below uses the following notation. Given integers $m \leq n$ and a sequence of numbers $a_m, a_{m+1}, \ldots, a_{n-1}, a_n$, then

$$\prod_{i=m}^{n} a_i = a_m \cdot a_{m+1} \cdot \cdots \cdot a_{n-1} \cdot a_n$$

3. In the Problems I which begin on page 53, do #17, 19, and 20. (#20 is challenging, but I think it is fun. My recommendation is to save it for last.)

4. In the Problems II which begin on page 115, do #1(part ii only), 2, and 3. (For #2, you may find it useful to write a truth table and you should definitely draw a Venn diagram. But I want you to treat these things as scratchwork, and instead write out an argument in words along the lines of the proof of Theorem 6.3.4 in the book. Your proof of #3 should also be in words along the same lines.)