

MATH 109 WINTER 2007 HOMEWORK 9, DUE 3/16/07 IN CLASS

(All exercise and page numbers refer to Eccles.)

1. READING AND PRACTICE

Read Chapters 12, 14 of the text, and do as many of the end of chapter exercises as possible.

2. EXERCISES TO SUBMIT ON FRIDAY 3/16

1. An *anagram* of a word is a way of rearranging its letters. For example, “and” is an anagram of “dan”. There are 6 possible anagrams of “dan”: namely, “dan”, “dna”, “nad”, “nda”, “and”, and “adn”.

(a). How many different anagrams of the word “number” are there? Justify your answer using our study of counting. (Don’t write them all down).

(b). How many different anagrams of the word “function” are there? Justify your answer by counting. (Hint: Note that this is harder than part (a) because of the appearance of two n’s. Think about the anagrams of the word “fuction”. How are the number of anagrams of “fuction” and the number of anagrams of “function” related?)

2. Recall the definition of a binomial coefficient: for $0 \leq n \leq r$,

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

Give a simple formula (i.e. not involving a sum of many terms) for the expression

$$\binom{n}{0} + \binom{n}{1} \cdot 4 + \binom{n}{2} \cdot 4^2 + \cdots + \binom{n}{n} \cdot 4^n$$

as a function of n . Justify that your answer is correct using the binomial theorem.

3. Every day a student in Manhattan walks from home to school, which is located 5 blocks east and 7 blocks north from home. She always walks to school in an efficient way, so the total walk takes 12 blocks, but she likes to take different routes. For example, one possible walk is to walk 5 blocks east and then 7 blocks north. Another possible walk is to walk 3 blocks east,

then 7 blocks north, then 2 blocks east. (Assume the streets in this part of Manhattan are a square grid. Drawing a picture may help.) How many different 12-block walks are possible? Explain your answer using our study of counting.

Hint: each block she walks along she is walking either east or north (she can not double back or go out of her way, because then the walk would take more than 12 blocks.) Represent each possible walk as a length-12 sequence of letters E and N in some order. Count the number of possible such sequences which represent a walk from her house to school.

4. (a). Read the proof that \mathbb{Q} is denumerable in your notes and study it until you understand it. (Note that the proof I gave in class was a little different from the one in the book.) Then put your notes away, and write out a proof here that \mathbb{Q} is denumerable *in your own words*. You may quote basic lemmas just as we did in class.

(b). Let $A = \{x + y\sqrt{2} \mid x, y \in \mathbb{Q}\}$. This is a subset of the set of real numbers, consisting of all linear combinations of 1 and $\sqrt{2}$ with rational coefficients. Prove that A is denumerable. (Hint: Find a surjective function from $\mathbb{Q} \times \mathbb{Q}$ to A .)

5. (a). Read the proof that \mathbb{R} is uncountable in your notes and study it until you understand it. (Note that the proof I gave in class was a little different from the one in the book.) Then put your notes away, and write out a proof here that \mathbb{R} is uncountable *in your own words*. You may quote basic lemmas just as we did in class.