Math 109 Spring 2006 HW 1

HW Due Wednesday 3/12/06 in class

Exercise and page numbers refer to Fletcher and Patty, 3rd edition. Check them against a friend's book if you have another edition.

Part 0

These are suggested exercises for extra practice if you feel you need it. Do not hand these exercises in.

FP, Chapter 1, #1, 2, 4, 6, 7, 15, 16, 23, 27, 41, 54, 56

Part 1

These are exercises which tend to be straightforward tests of your understanding. Turn all of these in, but they will not all be carefully graded.

FP, Chapter 1, #20, 36, 37, 53, 57, 59

Part 2

These are more challenging and/or longer exercises. They will all be graded.

1. In class, we defined the connectives $\vee$, $\wedge$, $\rightarrow$, and $\leftrightarrow$, and the negation symbol $\neg$. We defined all of these because they correspond to the most useful ways that two propositions can be combined in mathematics. But one can get away with a smaller set of connectives, in the sense that some of these can be defined in terms of the others. In fact, $\vee$ and $\neg$ are enough. For

example, we can define $\wedge$ in terms of $\vee$ and $\neg$ as follows: The proposition $P \wedge Q$ is logically equivalent to $\neg(\neg P \vee \neg Q)$.

(a). Prove that $P \wedge Q$ is logically equivalent to $\neg(\neg P \vee \neg Q)$, as claimed above.

(b). Find an expression which is logically equivalent to $P \rightarrow Q$, but which only uses $P, Q, \vee$ and $\neg$. Prove it is logically equivalent.

(c). Find an expression which is logically equivalent to $P \leftrightarrow Q$, but which only uses $P, Q, \vee$ and $\neg$, and again prove your answer.

2. Translate each of the following propositions into a sensible English sentence. Then decide if the proposition is true or not. Both variables range over real numbers.

(a) $(\forall y)(\exists x)[(x \leq y) \wedge (y \leq x^2)]$

(b) $(\exists x)(\forall y)[(x \leq y) \vee (y \leq x^2)]$

3. Give a direct proof of each of the following statements.

(a). For every odd integer n, $8|(n^2 - 1)$. (If this seems too hard, start by trying to prove that $4|(n^2 - 1)$.)

(b). Let a, b, c, and d be integers. If $a|b$ and $c|d$, then $(ac)|(bd)$.

4. Recall that for any positive integer n, $n!$ is defined to be the integer $1 \cdot 2 \cdot 3 \cdot \cdots \cdot (n-1) \cdot n$. For example, $1! = 1,\ 2! = 2,\ 3! = 6,\ 4! = 24$, and so on. Now give direct proofs of each of the following statements. (If you happen to have learned proof by induction before, do not use it.)

(a). For every positive integer n, if n is neither a prime number nor a perfect square, then n divides $(n-1)!$.

(b). For every positive integer n, $2(n!) \leq (2n)!$.