Math 109 Spring 2006 HW 2

HW Due Wednesday 4/19/06 in class

Exercise and page numbers refer to Fletcher and Patty, 3rd edition. Check them against a friend's book if you have another edition. Working with a classmate is fine, but the final writeup should be your own work.

Part 0

These are suggested exercises for extra practice if you feel you need it. Do not hand these exercises in.

FP, Chapter 1, #75, 77, 88, 92, 93, 94

FP, Chapter 2, #3, 7, 10, 11, 23, 25, 44, 47

Part 1

These are exercises from the book which you are to turn in, but they will not all be carefully graded.

FP, Chapter 1, #91, 99

FP, Chapter 2, #24, 30, 43

Part 2

These are more challenging and/or longer exercises.

1. (a) Let A and B be sets. Now prove that $A \subseteq B$ if and only if $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

(b). Prove that if $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$, then $A \subseteq B$ or $B \subseteq A$.

2. Prove that for any 3 sets A, B, and C, one has

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

3. Let U be some universe. A subset $A \subseteq U$ is called *cofinite* if the complement of A in U has finitely many elements.

Now prove: If A and B are cofinite subsets of U, then $A \cap B$ is also cofinite.

4. A *pythagorean triple* is a triple of positive integers (a, b, c) such that $a^2 + b^2 = c^2$. In other words, a set of three numbers is a pythagorean triple if there exists a right triangle having these three integers as its side lengths. The triples one most commonly encounters when one first studies right triangles are $(3, 4, 5)$ and $(5, 12, 13)$.

(a) Are there are other pythagorean triples besides $(3, 4, 5)$ which consist of three consecutive positive integers? Either give an example of another such triple, or else prove no other such triple can exist.

(b) Prove that there are infinitely many pythagorean triples (a, b, c) for which $c - b = 1$. (Suggestion: direct proof by constructing them explicitly. Start by finding some small ones and look for a pattern.)

(c) Prove that there does not exist a pythagorean triple (a, b, c) such that a is odd, b is odd, and c is even. (Suggestion: Assume (a, b, c) is such a triple and derive a contradiction.)

5. (a) Let n be an integer. Prove that if $3|n^2$, then $3|n$. (Suggestion: Prove the contrapositive.)

(b). Prove that $\sqrt{3}$ is irrational. (Hint: follow the example of the proof by contradiction given in class that $\sqrt{2}$ is irrational. Part (a) may be helpful, so you can assume part (a) even if you can't do part (a)).