Math 109 Spring 2006 HW 3

HW Due *Friday* 4/28/06 in class

Exercise and page numbers refer to Fletcher and Patty, 3rd edition. Check them against a friend's book if you have another edition. Working with a classmate is fine, but the final writeup should be your own work. *Starred problem numbers indicate problems you should make sure you understand before the first midterm.*

Part 0

These are suggested exercises for extra practice if you feel you need it. Do not hand these exercises in.

FP, Chapter 2, #55, 56, 57.

FP, Chapter 3, any induction proofs from the exercises for (3.1)-(3.3) are good.

Part 1

These are exercises from the book which you are to turn in, but they will not all be carefully graded.

FP, Chapter 2, #$53^*, 54^*, 68^*$.

FP, Chapter 3, #$1(i)^*(q)^*, 6^*, 22^*, 25$.

Part 2

These are more challenging and/or longer exercises.

1*. Prove by induction that the following formula holds for all $n \in \mathbb{N}$:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

2. Recall that the Fibonacci numbers are a sequence of numbers $f_0, f_1, f_2, \ldots$ which is defined as follows: $f_1 = 1$, $f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for all $n \geq 3$. Prove by induction that the following formulas hold for all $n \in \mathbb{N}$.

(a) (Binet's formula)

$$f_n = \frac{1}{\sqrt{5}} \left\{ \left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right\}.$$

(b)

$$(f_{n+1})^2 - f_{n+2} f_n = (-1)^n.$$

3. Prove that $3^n > n^3$ for all $n \in \mathbb{N}$ such that $n \geq 6$.

4. Prove the following statements by induction on n. (they can also be proved by other methods, but I want you to use induction.)

(a) Given any n Red Sox fans, there are $n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$ different possible orderings of them in a queue at the Fenway Park ticket window.

(b) Suppose a Boston radio station offers 2 free tickets to a Sox game. Given a set of n Red Sox fans who try for the tickets, there are $n(n-1)/2$ possible pairs of fans who win the tickets. (For example, if $n = 3$ and Joe, Karen, and Larry are the fans, then there are 3 possible pairs of winners: Joe and Karen, Karen and Larry, and Joe and Larry.)