Math 109 Spring 2006 HW 4

HW Due Wednesday 5/3/06 in class

Exercise and page numbers refer to Fletcher and Patty, 3rd edition. Check them against a friend’s book if you have another edition. Working with a classmate is fine, but the final writeup should be your own work.

1. FP, Chapter 3, #84(a), (c).

2. Prove that for any two integers \(a, b \in \mathbb{Z}\), not both 0, then \(\gcd(a, b) = \gcd(|a|, |b|)\). (Here \(|x|\) denotes the absolute value of the integer \(x\).)

3. FP, Chapter 3, #101. You may assume Corollary 3.13 from the book (which we also proved in class) in your proof.

4. Let \(a\) and \(b\) be two positive integers. A \textit{common multiple} of \(a\) and \(b\) is a positive integer \(m\) such that \(a|m\) and \(b|m\). The \textit{least common multiple} of \(a\) and \(b\), written \(\text{lcm}(a, b)\), is defined to be the smallest common multiple of \(a\) and \(b\) (which exists by the least-natural-number principle).

   (a). Prove that a positive integer \(d\) is a common divisor of \(a\) and \(b\) if and only if \(ab/d\) is an integer and \(ab/d\) is a common multiple of \(a\) and \(b\).

   (b). Using part (a), prove that \(\text{lcm}(a, b) = (ab) / \gcd(a, b)\).