

Math 109 Spring 2006 HW 4

HW Due Wednesday 5/3/06 in class

Exercise and page numbers refer to Fletcher and Patty, 3rd edition. Check them against a friend's book if you have another edition. Working with a classmate is fine, but the final writeup should be your own work.

1. FP, Chapter 3, #84(a), (c).

2. Prove that for any two integers $a, b \in \mathbb{Z}$, not both 0, then $\gcd(a, b) = \gcd(|a|, |b|)$. (Here $|x|$ denotes the absolute value of the integer x .)

3. FP, Chapter 3, #101. You may assume Corollary 3.13 from the book (which we also proved in class) in your proof.

4. Let a and b be two positive integers. A *common multiple* of a and b is a positive integer m such that $a|m$ and $b|m$. The *least common multiple* of a and b , written $\text{lcm}(a, b)$, is defined to be the smallest common multiple of a and b (which exists by the least-natural-number principle.)

(a). Prove that a positive integer d is a common divisor of a and b if and only if ab/d is an integer and ab/d is a common multiple of a and b .

(b). Using part (a), prove that $\text{lcm}(a, b) = (ab)/\gcd(a, b)$.