Math 109 Spring 2006 HW 5

HW Due Wednesday 5/10/06 in class

Exercise and page numbers refer to Fletcher and Patty, 3rd edition. Check them against a friend's book if you have another edition. Working with a classmate is fine, but the final writeup should be your own work.

Part 0

These are suggested exercises for extra practice if you feel you need it. Do not hand these exercises in.

FP, Chapter 3, #117.

FP, Chapter 4, #31, 44

Part 1

Turn these exercises in.

FP, Chapter 3, #116, 118(a).

FP, Chapter 4, #27, 28, 34, 35, 36, 37

Part 2

These are more challenging and/or longer exercises.

1. In a certain ancient Martian society, only two types of coins were minted, which had values of 11 qix and 15 qix, and there was no paper money.

(a). Most things on Mars at the time happened to cost 367 qix. Find all possible combinations of the two coins which would allow a Martian to pay for such an item with exact change.

(b). Martian panhandlers often asked passersby if they could spare 20 qix. Explain a way in which someone could give the panhandler exactly 20 qix. (Assume both the panhandler and the passerby have an adequate supply of both types of coins to begin.)

(c). Find the simplest way of giving the panhandler 20 qix (which means the way which uses the smallest total number of coins exchanging hands). Prove your answer is correct.

2. Prove that for all $n \in \mathbb{Z}$, $\gcd(5n+2, 12n+5) = 1$. (Hint: note that n is any integer, not just a positive integer. The presence of the letter n probably makes you think immediately of proof by induction, but that is not the way to go for this problem.)

3. This problem describes an equivalence relation important in algebra. Let $S = \{(a,b) | a, b \in \mathbb{Z}, b \neq 0\}$. In words, S is the set of ordered pairs of integers in which the second coordinate is not zero. Define a relation on the set S as follows. For (a,b) and (c,d) in S, define $(a,b) \sim (c,d)$ if and only if $ad = bc$.

(a). Prove that $\sim$ is an equivalence relation on S.

(b). Prove that given $(a,b) \in S$, the equivalence class $[(a,b)]$ consists exactly of the set of those ordered pairs $(c,d) \in S$ such that the fractions a/b and c/d are equal.

(c). Suppose you are given two equivalence classes $[(a,b)]$ and $[(c,d)]$ for this equivalence relation. Prove that for any $(v,w) \in [(a,b)]$ and $(x,y) \in [(c,d)]$, $(vx, yw) \in [(ac, bd)]$.

(d). In algebra, the ordered pair $(a,b) \in S$ is used as a kind of representation of the fraction a/b. The advantage of this is that the notion of two fractions being equal translates to two ordered pairs being in the same equivalence class, by part (b). Try to interpret the result of part (c) in terms of fractions.