Math 109 Spring 2006 HW 6

HW Due Wednesday 5/17/06 in class

Exercise and page numbers refer to Fletcher and Patty, 3rd edition. Check them against a friend's book if you have another edition. Working with a classmate is fine, but the final writeup should be your own work.

Part 0

These are suggested exercises for extra practice if you feel you need it. Do not hand these exercises in.

FP, Chapter 6, #3, 14, 15, 48, 53, 55

Part 1

Turn these exercises in. Many of these problems are computational, but you should justify your answers.

FP, Chapter 6, #4, 7, 20, 23, 35, 54

Part 2

These are more challenging and/or longer exercises.

1. Let $S = \mathbb{R}^2$ be the usual Cartesian plane, consisting of all points (a, b) with real coordinates. Define a relation $\sim$ from S to itself by defining $(a, b) \sim (c, d)$ to mean $a - c = 2b - 2d$.

(a) Prove that $\sim$ is an equivalence relation.

(b). Since $\sim$ is an equivalence relation, there is a corresponding partition of the the set S. Describe this partition geometrically. In other words, what "shape" do the equivalence classes making up the partition have in the plane? What does the fact that the equivalence classes are pairwise disjoint mean geometrically? Draw a picture to illustrate your answer.

2. Consider a standard deck of 52 cards with 4 suits each containing 13 cards, as in Example 7 on page 199 of the book (You should understand that example before beginning this problem). How many 13-card bridge hands are there which have a singleton in at least one suit? (Having a singleton in a suit means having exactly one card in that suit. By the way, having a short suit is often a good thing in bridge.) Your answer can be in terms of various $C(n,k)$, you don't have to evaluate these to get an actual number.

3(a). Using the binomial theorem, give a simple formula (not involving a sum of many terms) for the expression

$$\binom{n}{0} + \binom{n}{1} \cdot 4 + \binom{n}{2} \cdot 4^2 + \cdots + \binom{n}{n} \cdot 4^n$$

as a function of n.

(b). Similarly, find a formula (not involving a sum of many terms) for the expression

$$\binom{n}{0} - \binom{n}{1} \cdot 3 + \binom{n}{2} \cdot 3^2 - \cdots + (-1)^n \binom{n}{n} \cdot 3^n$$

as a function of n.

4. Again, in this problem your answers can be in terms of the numbers $C(n,k)$, you don't have to evaluate them. Every day a student in Manhattan walks from home to school, which is located 10 blocks east and 14 blocks north from home. She always takes a shortest walk of 24 blocks, but there are many different possible routes. (Assume the streets in this part of Manhattan are a square grid. Drawing a picture may help.)

(a). How many different walks are possible? (Hint: each block she walks along she is walking either east or north.)

(b). Suppose located 4 blocks east and 5 blocks north of her home is a street corner where a known drug dealer hangs out, so her mother has told her to avoid this intersection. How many different walks which do not pass through this intersection are possible? (Hint: Think first about how many walks there are which do pass through this intersection.)