

Math 109 Homework Suggestions

Now that the first homework is done and graded, here are some suggestions for improvement.

1. *Completing the homework.* This point is directed at those few of you that handed in homework with lots of blanks (a small minority.) You can't get a good score on the homework, and you will not learn all of the material you need for the class, if you don't make a serious effort to complete the homework. If you get stuck on the harder problems, ask a friend for advice, or ask in office hours or section for a hint; just remember that the final writeup must be your own work. The homework will take time—your brain needs time to figure these out, most of the problems you will not solve instantaneously— so start it early.

2. *Advice on the process.* How do you approach a proof oriented problem? Here is a suggested sequence of steps.

- First, make sure you understand exactly what the proposition you are trying to prove means. This step is very important, and becomes more and more important the more complicated the proposition is. Breaking this down even further, first make sure you understand the definitions of all terms and symbols in the proposition, looking them up in the book or your notes if you are not absolutely sure. Then, understand the logic of the proposition. If it is an if...then statement (the most common type), make sure you are clear on what the hypothesis is, and what the conclusion is. Keep the two clearly separate in your head, so you know what you are assuming, and what you are trying to prove from that assumption.

- Try to get some feel for why the proposition might be true. For example, if the proposition is a statement about integers, replace the variables by

some small values and see what the proposition says for those small numbers. This is the “investigation” stage that has been mentioned in class. Look for a pattern.

- This is the most nebulous stage, and the hardest to give advice for. You have to turn your intuition or the pattern you gathered in the investigative stage into a proof. See if you can find a sequence of arguments going directly from the hypothesis to the conclusion (direct proof). If starting with the negation of the conclusion, you can find a sequence of arguments going directly to the negation of the hypothesis, this is also good (proof by contrapositive). See if assuming something you expect to be false (such as the negation of the conclusion), you can derive a contradiction (proof by contradiction.) If the statement involves a variable ranging over natural numbers, proof by induction might be possible (we will learn this soon.)

- Once you *think* you have found an argument, on scratch paper, try to write it out. Make sure you make a clear choice as to what type of proof you are doing. As you write out your proof, you may notice there are minor points in your argument you didn't think carefully enough about before. Because this is a draft, you can add carets, notes in the margin, and cross things out to your heart's content, and you need not worry about writing in complete sentences. This step is also extremely important, because it is very common that you will not see any flaws in your argument until you try to write the proof down. This is the time to fix them. Once you think you are done, read over your proof to make sure the logic is correct and you still believe it.

- Now you have convinced yourself that you have a good proof, but it is probably messy and unreadable. The next step is to write your proof in a way that will convince others (especially the grader.) So make a clean copy on to the homework paper you will submit. Do not include the investigative work or any scratch work. Write in complete English sentences. Your notation should be the same as the notation introduced in class. Now you have an exemplary proof!

3. The most common problems on HW#1.

- Style is important. We are serious when we ask you to write in complete sentences, and to hand in neat and clean homework that is not a first draft. The reason for the complete sentences is simply that it makes your proof

easier to read and understand, and it makes the logical structure of the proof more clear. Many of you handed in first drafts of your HW#1. Avoid carets and scratch outs. Your work should simply flow top to bottom on the page. If you are using arrows to tell the reader where to look next, you are still on draft #1.

- In some cases, even if you have the “right idea”, your proof will not be convincing if you do not make the logical structure clear. Tell us if it is a direct proof, proof by contrapositive, or proof by contradiction. Clearly state what the hypotheses are. Make clear how each line follows from the preceding lines. Many of the mistakes on HW#1 can be traced to unclear logical structure. For instance, there were many papers which assumed the desired conclusion at the beginning of the proof, which is certainly not allowed. In some cases, a lot of the steps of a correct proof were present, but the proof was more or less presented backwards. In a direct proof (which every proof on HW#1 was supposed to be), start with the hypothesis, and argue line by line to the conclusion. Starting with the conclusion and arguing towards the hypothesis is not a valid proof (because this is proving the converse of the proposition, which is a different statement altogether.)

- Use lined or blank paper (it is hard to read writing on graph paper.) The grader mentioned that it would help speed up the grading if you only wrote on one side of each page—if you are happy with doing this, great, but if it taxes your paper resources too much, OK.

- Most of you are seriously on the right track. Just keep up the hard work, but keep the points above in mind as you write the next homework.