Math 200a Fall 2008 Exam 1

November 1, 2008

Do not be concerned by the length of the exam. Just do as many of the problems as well as you can and don't worry if you don't finish everything. You may quote results proved in class or in the textbook, but try to avoid quoting results proved only in homework exercises.

NAME:

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

1. Let G be the group given by the presentation

$$G = \langle a, b | a^2 = e, b^2 = e, (ab)^3 = e \rangle.$$

Prove that G is isomorphic to a very familiar finite group.

2. The *category of politically colored sets* $\mathcal{C}$ is defined as follows. An object of $\mathcal{C}$ is a pair (X, f), where X is a set and $f : X \to \{blue, red\}$ is a function. (Intuitively, you should think of f as a fixed coloring of the elements of X with the colors red and blue.) The set of morphisms from (X, f) to (Y, g) consists of all functions $\phi : X \to Y$ which preserve color, in other words such that $\phi(x) \in Y$ has the same color as $x \in X$ for all x. Composition of morphisms is composition of functions. It is easy to see this satisfies the definition of a category; do not prove this.

First recall the definition of the *product* of two objects in the category $\mathcal{C}$. Then show by explicit construction that the product of any two objects exists in this category. (Do this only for two objects as suggested, not an arbitrary family.)

3. Let n be an integer with $n \geq 5$. Show that S_n has *exactly one* proper normal subgroup, freely using any results about permutation groups from the book or from class.

(Hint: what is special about A_n for $n \geq 5$? Use this fact in your proof.)

4. Let G be a finitely generated group. Show that as long as $|G| \geq 3$, then the automorphism group of G, $\operatorname{Aut} G$, contains at least two elements.