Math 200a Fall 2011 Homework 1

Due Friday 9/30/2011 by 5pm in homework box

Reading assignment: Review Chapters 1-3 of Dummit and Foote, and read Section 6.3. I suggest you go carefully over the proofs of the isomorphism theorems in Section 3.3.

Exercises:

All exercise numbers refer to Dummit and Foote, 3rd edition. I like to list extra exercises from the text, but *only the exercises marked with a star are to be handed in for grading.* I include these extra exercises because they seem interesting or useful but I cannot include them without having too many assigned exercises. So please at least look over the unstarred exercises. I also frequently assign some exercises not from the text; usually these are all starred and thus to be handed in.

Section 1.1 #25*, 31

Section 1.3 # 15*, 16, 17, 19*

Section 1.6 #6, 7, 18, 25, 26

Section 2.1 #6, 7

Section 2.4 # 14, 18*(c,d only),19

Section 3.1 #5, 12, 14*(c,d only) (suggestion for (d): prove using the homomorphism of exercise 12 that $\mathbb{R}/\mathbb{Z}$ is isomorphic to the circle group S^1, i.e. the subgroup of $\mathbb{C}^*$ consisting of elements of norm 1, then look at the torsion subgroups of both sides), 15, 36*, 42

Section 3.2 #9*, 20*, 21

Section 3.3 #3, 7* (suggestion: use the first isomorphism theorem).

Exercises not from the text: (all to be handed in):

1*. Suppose that $G = H \cup K$, where H and K are subgroups of G. Show that either $G = H$ or $G = K$ (i.e., no group is the union of two proper subgroups.)

2*. Suppose that G is a finite group and that $G = H_1 \cup H_2 \cup H_3$, where each H_i is a proper subgroup of G. Show that $|G : H_i| = 2$ for all i. Also, find an example where this actually happens. (Hint: first show by counting that at least one of the subgroups, say H_1, has index 2. Then prove that this forces $H_1 H_i = G$ and $|H_i : H_1 \cap H_i| = 2$, for $i = 2, 3$, and do some more counting.)