Math 200a Fall 2011 Homework 2

Due Friday 10/7/2011 by 5pm in homework box on 6th floor of AP&M

Reading assignment: Section 6.3 (if haven't finished), 4.1-4.3.

Exercises:

All exercise numbers refer to Dummit and Foote, 3rd edition. I like to list extra exercises from the text, but *only the exercises marked with a star are to be handed in for grading.* I include these extra exercises because they seem interesting or useful but I cannot include them without having too many assigned exercises. So please at least look over the unstarred exercises. I also frequently assign some exercises not from the text; usually these are all starred and thus to be handed in.

Section 6.3 #1, 4, 7*, 11*

Section 4.1 #1, 2*, 3*, 7*, 8, 9*

Section 4.2 #7*, 8*, 10, 14

Exercises not from the text: (all to be handed in):

The property "is a normal subgroup of" is not transitive in general; that is, if one has $H \trianglelefteq K$ and $K \trianglelefteq G$, it does not follow in general that $H \trianglelefteq G$. Recall that an *automorphism* of a group G is an isomorphism from G to itself. We say that a subgroup $H \leq G$ is *characteristic* in G, and write $H \operatorname{char} G$, if for all automorphisms ϕ of G, $\phi(H) = H$.

1.* (a). Show that if $H \operatorname{char} G$, then $H \trianglelefteq G$.

(b). Let $H < K < G$, where $H \operatorname{char} K$ and $K \trianglelefteq G$. Then $H \trianglelefteq G$.

(c). Show that if K is a cyclic subgroup of G and $K \trianglelefteq G$, then every subgroup H of K satisfies $H \trianglelefteq G$.

2.* Let $f, g : \mathbb{R} \to \mathbb{R}$ be the functions defined by the formulas $f(x) = -x$ and $g(x) = x+1$. Let $G = \langle f, g \rangle$ be the subgroup of the group $S_{\mathbb{R}}$ of all permutations of the set $\mathbb{R}$ which is generated by f and g. Prove carefully that

$$G \cong \langle a, b \,|\, b^2 = e, ba = a^{-1}b \rangle.$$

(This group is called the *infinite dihedral group*, A_∞.)